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ABSTRACT

This thesis presents a computer program designed to estimate the first seven natural frequencies and their respective modes for ships and to estimate ship response in a Pierson-Moskowitz sea. The program was designed to function on a minimum of input. Hull offsets and deck edge height for each station are the major data required. The program computes the hydrodynamic added mass, the polar moment of inertia J, the second moment of area I, the effective shear stiffness KAG and the ship mass for each station. A Prohl sequence is used to determine hull natural frequencies and mode shape. A wave forcing function is presented which sums buoyancy, added mass and damping factors due to wave passage. The program can be used for all vessels, but program development was based on a single beam model so superstructure effects are not considered. A minimum of 20 stations is required.

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VIBRATION OF SHIP HULLS DUE TO WAVE EXCITATION

by

SCOTT TEMPLE SMITH

B.S.M.E., University of Arizona
(1972)

Submitted in Partial Fulfillment
of the Requirements for the
Degrees of

OCEAN ENGINEER

and

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

at the

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VIBRATION OF SHIP HULLS DUE TO WAVE EXCITATION

by

SCOTT TEMPLE SMITH

Submitted to the Department of Ocean Engineering on May 11, 1979, in partial fulfillment of the requirements for the degrees of Ocean Engineer and Master of Science in Mechanical Engineering.

ABSTRACT

This thesis presents a computer program designed to estimate the first seven natural frequencies and their respective modes for ships and to estimate ship response in a Pierson-Moskowitz sea. The program was designed to function on a minimum of input. Hull offsets and deck edge height for each station are the major data required. The program computes the hydrodynamic added mass, the polar moment of inertia J, the second moment of area I, the effective shear stiffness KAG and the ship mass for each station. A Prohl sequence is used to determine hull natural frequencies and mode shape. A wave forcing function is presented which sums buoyancy, added mass and damping factors due to wave passage. The program can be used for all vessels, but program development was based on a single beam model so superstructure effects are not considered. A minimum of 20 stations is required.

Supervisor: Joao Manuel Gomes de Oliveira
Title: Assistant Professor of Ocean Engineering

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1. INTRODUCTION

Vibration is a phenomenon specially appealing to a number of engineers due partly to a clearly defined, extensive and intriguing body of underlying mathematical analysis. From the late 1800's when attention was first paid to the problem of ship hull vibration this body of analysis has been expanded by a large number of active researchers to provide the current comprehensive level of understanding of the subject.

It is apparent that vibration of the ship hull is a source of concern for the naval architect (and shipowner). A passenger vessel with vibration amplitudes above a certain threshold would have few repeat passengers. Fatigue of structural components due to the cyclic stresses is a factor to be considered. The conventional method of determining the strength of the hull girder does not explicitly take into account the cyclic stress associated with hull vibration. It would seem necessary to determine what these stresses might be, how they vary with time and the effect of their addition to stresses already present through ship loading and the quasistatic effect of wave support. The sophisticated equipment on board warships experience problems in an environment of excessive vibration. It is interesting to note that with continued refinement of such equipment as navigation instruments, radar, sonar, fire control computers, communications gear and electronic warfare devices the requirement for

vibration free foundations becomes more stringent. There are dangers to shipping from acoustic and pressure operated offensive weapons. Attention has been focused on noise emitted by hulls and hull vibration in general. The "acoustic signatures" of vessels play an important part in tracking foreign vessels, both surface and subsurface, by passive means. It appears that the problem of vibration control is becoming more important.

As stated, the body of analysis is large but it is apparent that the complexity of hull structures in both form, material and construction and the complexity of three-dimensional hydrodynamics combine to limit the application of the analysis to reality. It is not surprising then that the problems of hull vibration still remain and that work continues on new methods of mathematical treatment, on models and on full size ships where the adequacy of the technical progress can be shown.

This thesis is concerned with one of the two broad classes of ship hull vibration, that is synchronous or resonant vibration as opposed to local vibration. In this case the entire hull acts like a beam vibrating under some excitation from machinery, the propellor or waves. In serious cases the motion can clearly be seen by sighting along the length of the ship. The importance of a resonant vibration is dependent on the conditions of its occurence.

If the engine at normal operating speed excites the hull then a serious problem exists. If, however, this resonance occurs at an engine speed that will be only infrequently maintained, then there is a problem of considerably less significance. Similarly, if a vessel is expected to operate in certain known sea states then it is necessary to avoid an excessive resonant condition by proper hull design.

The other class of hull vibration is a local effect. Here specific parts of the ship or certain fittings such as masts, stacks, superstructure or a panel of plating are set into motion. Depending on the point of view local vibration can also include the undesired motion of items as small as light fixtures or stanchion chains. Local vibration can have the same effects as hull resonance, that is personal discomfort and structural and equipment failures. This type of vibration is not as readily predicted and analyzed as whole hull vibration and is treated generally after the fact by locating and removing the excitation or adding local stiffening, pillars and similar structures. Obviously in many cases the excitation cannot be eliminated and additional structure is not feasible. Then the solution is the removal of the object or relaxation of the requirements.

It is apparent that the hull of a ship will vibrate only if an external force is applied to it. When this force is from slamming, the resulting shudder can be picked up by

accelerometers and can be shown to be transient in nature. A similar force can be generated by rapidly releasing a haused anchor or rapidly braking a free falling one. The transient response and information which can be obtained from it will be discussed later. Other sources of excitation can produce continuous vibration which is somewhat more insidious due to its gradual deleterious action. Such sources include those whose origin is some out-of-balance periodic force in the diesel main engine, auxiliary machinery, shafting or propellers.

It can therefore be reduced in any given instance by proper attention to balancing during construction. Diesel engine balancing is a well developed art which depends on the type of engine, number of cylinders, auxiliaries which may be run from the main engine and many other factors. Properly operating turbine equipment runs at angular velocities well above the frequencies of concern here.

The size of shafting and propellers makes perfect balancing impossible. In addition other once per revolution forces can be created if one blade of the propeller has a different shape than the other blades. Even with a perfect propeller, the blades work in a mixed wake behind the hull and the force on each varies continuously throughout each revolution. These varying forces are conveniently divided into two types according to the method of transmission to the hull.

"Propeller" forces are transmitted through the shafting to struts and bearings. "Surface" forces are transmitted through the water to the stern. Both types have a frequency of propeller angular velocity times the number of blades and are referred to as blade-rate forces. Extensive work has been carried out and resultantly the magnitude of these forces can be calculated quite accurately. For example, the propeller forces can be determined from lifting surface theory assuming the hull induced velocity field in the propeller plane is known. With the requirement that the fluid particles should remain in contact with the propeller blades, it follows that the velocity components normal to the surface of the blades due to the hull induced wake should be equal to the corresponding components of the propeller induced fluid velocities. This requirement can be expressed as a three-dimensional integral equation. The numerical computation is formidable but computer solutions yield the pressure distribution and the blade lift. Integration yields the forces. It is interesting to note that due to the dynamic behavior of the shaft, bearing reactions can be an order of magnitude higher than the forces generated at the propeller. All of the forces can be kept within tolerable limits by careful manufacture and proper attention to stern and appendage design and particularly to blade tip clearance. Some hydrodynamic disturbing forces have to be accepted, however, as part of the price of mechanical propulsion.

The excitation due to wave passage is at present an active area of research. The circular frequency of encounter of waves, ω_e, is dependent on the wave spectrum of the sea surface and the speed of the vessel. Even for the swiftest of destroyers the amount of energy available from excitation of even the lowest natural frequency is very small, the preponderance of wave energy being at lower frequencies. Ships of more recent vintage, however, are longer with some lengths over 1,000 feet and natural frequencies for the two node mode of between 3 and 5 radians/sec. There is ample energy for excitation in this range and numerous studies have concluded that "springing" as two node vibration is called is a significant addition to midship stress levels. [1][12] [16][30]

When engines were first put on ships hull vibration problems arose. Shortly thereafter an effort was made to predict the frequency for the two node mode of vibration. As a result of very early work by Otto Schlich and by F.W. Todd, empirical formulae were developed which closely correlated with observed frequencies. Schlich proposed a modification to the natural frequency formula for a uniform free-free beam which is

$$\omega_2 = \frac{(\beta_2 L)^2}{2\pi} \sqrt{\frac{EI}{mL^4}} \qquad (1)$$

where $\beta_2 L = 4{,}730$, E is the Young's modulus, I is the section moment of inertia, m is the beam mass per unit length, and L is the length. This formula will be explained more fully later. Then

$$\omega_2 = N \sqrt{\frac{I}{\Delta L^3}} \tag{2}$$

where the units are tons for Δ, feet for L, ft^4 for I, and radians/sec for ω_2. Schlick gave the following values for N:

ships with fine lines.....1369.
passenger liners.....1252.
cargo ships.....1116.

Another early effort which was twin steps in the directions of simplicity and accuracy by F.W. Todd was based on

$$I = CBD^3 \tag{3}$$

where B is the beam, D is the depth, and C a coefficient dependent on L/D and the ship type. In the Todd formula, the virtual mass was used which was a function of the beam to draft ratio B/T. So

$$\omega_2 = \beta \sqrt{\frac{BD^3}{\Delta_1 L^3}} \tag{4}$$

where

$$\Delta_1 = \Delta(1.2 + B/3T) \tag{5}$$

Values for β are listed in [32].

In the treatment which follows the natural frequencies of a vessel are determined from offset data and other information. The process is an iterative one and can be expedited by an appropriate choice of a starting frequency. The choice of this value is from a formula devised by L.C. Burrill.

$$\omega_2 = 80{,}000\pi \left[\frac{I}{\Delta L^3 (1+B/2T)(1+r_s)}\right]^{1/2} \tag{6}$$

where

$$r_s = \frac{3.5D^2[3(B/D)^3+9(B/D)^2+6(B/D)+1.2]}{L^2(3B/D+1)} \tag{7}$$

The Taylor shear correction term r_s is dimensionless.

The analysis of hull vibration has proceeded far beyond that which produced the preceding empirical formulations and this work attempts to explore the advances in considerations of damping, excitation, added mass and the use of the computer.

2. TECHNICAL TREATMENT

A. Uniform Beam Vibration

As previously stated, the mathematical base for the analysis of vibrating bodies is large and acceptance of certain broad assumptions allows complex problems to be idealized and working solutions obtained. As more realistic assumptions are made more intimate knowledge of the process is obtained. The price, of course, is vastly more complex solution techniques.

Consider a beam of length L. The vertical displacement at any time, t, is $y(x,t)$ where x is the longitudinal axis of the beam. The total vertical force per unit length is $f(x,t)$. The system parameters are the mass per unit length $m(x)$ and the flexural rigidity $EI(x)$ where E is Young's modulus of elasticity and $I(x)$ is the cross section area moment of inertia about the neutral axis of bending at any position x along the beam. Consider an element of length dx of the beam. $V(x,t)$ and $M(x,t)$ are the shearing force and bending moment. The simplifications to be included at this point are 1) shear deformation is small in relation to bending deformation and 2) rotary inertia effects are small.

These simplifications are valid if the beam's length to depth ratio L/D is about 10 or greater and if deflections and slopes are small. Most ships underway fall into this category. [14] Another major assumption is that damping is

small enough to be neglected. This is sufficiently accurate for vibrating ships particularly in the free vibration analysis which follows. [12] The small damping present serves to limit vibration amplitude rather than modify frequency or vibration pattern. The force equation of motion in the vertical direction is then

$$\left[V(x,t)+\frac{\partial V(x,t)}{\partial x}\,dx\right] - V(x,t)+f(x,t)\,dx=m(x)\,dx\frac{\partial^2 y(x,t)}{\partial t^2} \tag{1}$$

The moment equation of motion, ignoring the inertia torque associated with the element rotation is

$$\left[M(x,t) + \frac{\partial M(x,t)}{\partial x}\,dx\right]-M(x,t)+\left[V(x,t)+\frac{\partial V(x,t)}{\partial x}\,dx\right]dx$$

$$+f(x,t)\,dx\,\frac{dx}{2} = 0 \tag{2}$$

Here we assumed that the elemental length dx is small enough so that f(x,t) is constant along the element. Equation (2) reduces to

$$\frac{\partial M(x,t)}{\partial x} + V(x,t) = 0 \tag{3}$$

Using (3) in (1) and simplifying results in

$$-\frac{\partial^2 M(x,t)}{\partial x^2} + f(x,t) = \frac{m(x)\,\partial^2 y(x,t)}{\partial t^2} \tag{4}$$

From basic strength of materials it is known that

$$M(x,t) = EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} \tag{5}$$

(5) into (4) yields

$$-\frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}]+f(x,t) = m(x)\frac{\partial^2 y(x,t)}{\partial t^2} \tag{6}$$

which is the differential equation for the flexural vibration of a beam. [25] This is a fourth order linear nonhomogenous partial differential equation. General solution theory requires a number of boundary conditions equal to the order of the equation. Since this equation must be integrated four times to get a solution, four arbitary constants are introduced, thus the requirement for four boundary conditions. In the case of interest the beam ends are free. Thus the shear and moment at the ends equal zero. From equations (3) and (5)

$$M(x,t) = EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} = 0 \tag{7}$$

$$V(x,t) = \frac{\partial}{\partial x}[EI(x) \frac{\partial^2 y(x,t)}{\partial x^2}] = 0 \tag{8}$$

$$x = 0, L$$

The next analytical step is consideration of free vibration. The equation of motion reduces to

$$-\frac{\partial^2}{\partial x^2}[EI(x)\ \frac{\partial^2 y(x,t)}{\partial x^2}] = m(x)\ \frac{\partial^2 y(x,t)}{\partial t^2} \tag{9}$$

with the boundary conditions (7) and (8). This forms a classic "boundary value" problem. It is opportune at this point to explore the possibility of synchronous motion. This is a motion in which the general shape of the beam displacement does not change with time. That is to say, every point of the beam executes the same motion, passing through the equilibrium position and the point of maximum excursion at the same time. This implies that the deformation g(x,t) is separable in space and time. Therefore a possible solution of this boundary value problem is of the form

$$y(x,t) = Y(x)F(t) \tag{10}$$

where Y(x) represents the general beam configuration in vertical vibration and depends on x alone, and where F(t) indicates the type of motion the beam executes and depends on t alone. Introducing (10) into (9)

$$\frac{-1}{Y(x)m(x)}\ \frac{\partial^2}{\partial x^2}\ [EI(x)\frac{\partial^2 Y(x)}{\partial x^2}] = \frac{1}{F(t)}\ \frac{\partial^2 F(t)}{\partial t^2} \tag{11}$$

where partial deriviatives have been replaced by total derivatives. In addition, the variables have been separated so that the left hand side depends on x along and the right hand side on t alone. It is apparent, therefore, that both sides must equal a constant. Let λ be that constant, then

$$\frac{d^2F(t)}{dt^2} - \lambda F(t) = 0 \tag{12}$$

where, since $Y(x)$, $m(x)$ and $I(x)$ are real, λ is real.

Let $F(t)$ be of exponential form

$$F(t) = Ae^{st} \tag{13}$$

then substitution of (13) into (12) yields

$$As^2e^{st} - \lambda Ae^{st} = 0$$

or on dividing by Ae^{st} (for $Ae^{st} \neq 0$)

$$s^2 - \lambda = 0$$

so

$$s = \pm\sqrt{\lambda} \tag{14}$$

If λ is a positive number then the two roots (14) are both real and equal but opposite in sign. This suggests that there are two solutions for F(t), one increasing and one decreasing exponentially with time. Both of these solutions are inconsistent with an undamped conservative system so the possibility of λ being positive can be discarded. Letting $\lambda = -\omega^2$ we have $s = \pm i\omega$ and

$$F(t) = A_1 e^{i\omega t} + A_2 e^{-i\omega t} \tag{15}$$

Expansion of this expression where

$$e^{i\omega t} = \cos\ \omega t + i\ \sin\ \omega t$$

yields

$$F(t) = A_1 \cos\ \omega t + iA_1 \sin\ \omega t + A_2 \cos\ \omega t - iA_2 \sin\ \omega t$$

$$= (A_1 + A_2)\cos\ \omega t + i(A_1 - A_2)\sin\ \omega t$$

This must reduce to a real expression and will if

$$A_1 + A_2 = C\cos\phi$$

$$i(A_1 - A_2) = C\sin\phi$$

then

$$F(t) = C\cos(\omega t - \phi) \tag{16}$$

Proceeding with the other part of (11) yields

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2Y(x)}{dx^2}\right] = \omega^2 m(x)\ Y(x) \tag{17}$$

$$0 < x < L$$

No general closed form solution of (17) exists. However, for certain special cases solutions can be obtained and for illustrative purposes the following major simplification will be made, EI(x) and M(x) are made constant along the beam's length. This takes all ships with the possible exception of some box shaped barges out of consideration but the information derived from such a step will provide some weight into the problem. (17) reduces under these assumptions to

$$\frac{d^4Y(x)}{dx^4} - \beta^4 Y(x) = 0 \qquad \beta^4 = \frac{\omega^2 m}{EI} \tag{18}$$

The general solution of (18) can readily be verified to be

$$Y(x)=A_1\sin\beta x+A_2\cos\beta x+A_3\sinh\beta x+A_4\cosh\beta x \tag{19}$$

Simplifying the boundary conditions (7) and (8) yield

$$\frac{d^2Y(x)}{d^2x} = \frac{d^3Y}{dx^3} = 0 \qquad x = 0,L$$

The second and third derivatives of Y(x) are

$$Y''(x)=A_1\beta^2\sin\beta x-A_2\beta^2\cos\beta x+A_3\beta^2\sinh\beta x+A_4\beta^2\cosh\beta x \quad (20)$$

$$Y'''(x)=A_1\beta^3\cos\beta x+A_2\beta^3\sin\beta x+A_3\beta^3\cosh\beta x+A_4\beta^3\sinh\beta x \quad (21)$$

At x=0 (20) and (21) yield

$$A_2 = A_4 \quad \text{and} \quad A_1 = A_3 \quad (22)$$

At x=L substitution into (20) yields

$$A_1(\sinh\beta L-\sin\beta L)+A_2(\cosh\beta L-\cos\beta L)=0 \quad (23)$$

and substitution into (21) yields

$$A_1(\cosh\beta L-\cos\beta L)+A_2(\sinh\beta L+\sin\beta L)=0 \quad (24)$$

Parenthetically, these two equations in two unknowns can be reduced to one equation leaving A_2 in terms of A_1

$$A_2 = \frac{-A_1(\cosh\beta L-\cos\beta L)}{\sinh\beta L+\sin\beta L} \tag{25}$$

Inserting (25) and (22) into (19) yields

$$Y(x)=A[\sin\beta x+\sinh\beta x-\frac{(\cosh\beta L-\cos\beta L)}{\sinh\beta L+\sin\beta L}(\cos\beta x+\cosh\beta x)] \tag{26}$$

From linear algebra the determinant of the coefficients in (23) and (24) must equal zero for non-trivial values of A_1 and A_2 to exist. That is

$$\begin{vmatrix} \sinh\beta L-\sin\beta L & \cosh\beta L-\cos\beta L \\ \cosh\beta L-\cos\beta L & \sinh\beta L+\sin\beta L \end{vmatrix} = 0$$

This is known as the "characteristic determinant" and reduces to

$$(\sinh\beta L-\sin\beta L)(\sinh\beta L+\sin\beta L)-(\cosh\beta L-\cos\beta L)^2 = 0$$

After simplification this yields

$$\cosh\beta L\cos\beta L = 1 \tag{27}$$

This is known as the "characteristic equation" and it is apparent that it can be solved for an infinite number of values of β, that is, for β_r where r=1,2,3... which are defined by

$$\beta_r^4 = \omega_r^2 \, m/EI \tag{28}$$

From (28) an infinite set of frequencies, ω_r, are obtained. These are natural frequencies. For each ω_r equation (26) yields a prescribed vibration shape $Y_r(x)$ known as a "natural mode."

The general problem of determining the values of β in (18) for which nontrivial solutions $Y(x)$ exist where certain boundary conditions are prescribed is called the "characteristic-value" or "eigen-value" problem with the parameters ω_r being the eigenvalues and the functions $Y_r(x)$ being eigenfunctions. [19][25] It is to be noted that the eigenfunctions are precise as to configuration but not to absolute value. Since (18) is homogeneous and $Y_r(x)$ is a solution, then so is $\alpha Y_r(x)$ where α is a constant multiplier. A unique property of these eigenfunctions forms the basis for this entire train of analytical thought. For the general case of beam vibration as expressed in (6) this property known as "orthogonality" is expressed by

$$\int_0^L m(x) \, Y_r(x) \, Y_s(x) = 0 \qquad r \neq s \tag{29}$$

This orthogonolity is with respect to the mass function which serves as a weighting term.

This orthogonality implies that any possible configuration of the beam can be represented as a linear combination of the natural modes of the beam. That is

$$y(x) = \sum_{r=1}^{\infty} C_r Y_r(x) \tag{30}$$

This is true for any and all time t so

$$y(x,t) = \sum_{r=1}^{\infty} \eta_r(t) Y_r(x) \tag{31}$$

where the C_r have been replaced by the time functions $\eta_r(t)$ which are known as the "natural coordinates."

As previously stated the eigenfunctions are not determined in regards to absolute value. A convenient normalization scheme is such that the eigenfunctions satisfy

$$\int_0^L m(x) Y_r(x) Y_s(x) = \delta_{rs} \qquad r,s=1,2,3... \tag{32}$$

where δ_{rs} is the dirac delta function

$$\delta_{rs} = \begin{cases} 0 & r \neq s \\ 1 & r = s \end{cases}$$

In the uniform beam example insertion of one eigenfunction and eigenvalue into (18) yields

$$EI \frac{d^4 Y_r(x)}{dx^4} = \omega_r^2 \, m \, Y_r(x) \tag{33}$$

Multiplying both sides by $Y_s(x)$ and integrating over the length gives

$$\int_o^L Y_s(x) \, EI \frac{d^4 Y_r(x)}{dx^4} = \int_o^L \omega_r^2 \, m \, Y_s(x) \, Y_r(x)$$

$$= \omega_r^2 \, \delta_{rs} \tag{34}$$

Now substituting the assumed solution (31) into the beam equation (9) and assuming a uniform beam yields

$$\sum_{r=1}^{\infty} \ddot{\eta}_r(t) \, m \, Y_r(x) + \sum_{r=1}^{\infty} q_r(t) \, EI \frac{d^4 Y_r(x)}{dx^4} = 0 \tag{35}$$

Multiplying through by $Y_s(x)$ and integrating over the length and considering (32) and (34) yields the independent set of ordinary differential equations

$$\ddot{\eta}_r(t) + \omega_r^2 \, \eta_r(t) = 0 \qquad r=1,2,3... \tag{36}$$

The solution to each of these equations can be shown to be

$$\eta_r = C_r \cos(\omega_r t - \phi_r) \tag{37}$$

where C_r and ϕ_r are determined from initial conditions of the beam, $y(x,o)$ and $\dot{y}(x,o)$.

Using (37) and (31) and setting t=0 yields

$$y(x,o) = \sum_{r=1}^{\infty} C_r \cos\phi_r \, Y_r(x) \tag{38}$$

$$\dot{y}(x,o) = \sum_{r=1}^{\infty} tC_r\omega_r \sin\phi_r \, Y_r(x)$$

multiplying by $mY_s(x)$ and integrating over the length yields

$$C_r\cos\phi_r = \int_0^L mY_r(x) \, y(x,o) \, dx$$

$$C_r\sin\phi_r = \int_0^L \frac{m}{\omega_r} Y_r(x) \, \dot{y}(x,o) \, dx \tag{39}$$

which can be solved for C_r and ϕ_r.

The free vibration case for the uniform free-free beam has now been completely solved. The natural modes $Y_r(x)$ and the natural frequencies ω_r have been computed. It is the orthogonality of these modes and the fact that they span the vector space in which they are contained that makes the expansion theorems (30) and (31) correct. The natural coordinates $\eta_r(t)$ can be determined from initial conditions.

The forced vibration case is slightly more complex. Here substitution of the assumed solution (31) into the beam equation (6) yields

$$\sum_{r=1}^{\infty} \ddot{\eta}_r(t) m Y_r(x) + \sum_{r=1}^{\infty} q_r(t)\ EI \frac{d^4 Y_r(x)}{dx^4} = f(x,t) \qquad (40)$$

By multiplying by $Y_s(x)$ and integrating over the length and considering the normalization integrals (32) and (34) the following independent set of ordinary differential equations are obtained

$$\ddot{\eta}_r(t) + \omega_r^2 \eta_r(t) = F_r(t) \qquad (41)$$

where

$$F_r(t) = \int_0^L Y_r(x)\ f(x,t)\ dx \qquad (42)$$

The solution to (41) generally involves the use of the convolution integral

$$\eta_r(t) = \frac{1}{\omega_r} \int_0^t F_r(t)\ \sin\ \omega_r(t-\tau)d\tau$$

$$+ \eta_r(0)\ \cos\ \omega_r t + \dot{\eta}_r(0)\ \sin\ \omega_r t \qquad (43)$$

where $\eta_r(0)$ and $\dot{\eta}_r(0)$ are as determined by (39).

This completes the classical analysis of the forced vibration of the free-free uniform beam. It is not difficult to see that significant complications arise when the desire is present for a closer representation of reality. In (7)

inclusion of the beam theory damping term results in

$$M(x,t) = EI(x)\frac{\partial^2 y(x,t)}{\partial x^2} + \beta(x)\frac{\partial^3 y(x,t)}{\partial x^2 \partial t} \qquad (7a)$$

where $\beta(x)$ represents viscous structural damping. More on this in Section E. The equation of motion (6) then becomes

$$m(x)\frac{\partial^2 y(x,t)}{\partial t^2} + \frac{\partial^2}{\partial r^2}\left(EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right) + \frac{\partial^2}{\partial x^2}\left(\beta(x)\frac{\partial^3 y(x,t)}{\partial x^2 \partial t}\right)$$

$$= f(x,t) \qquad (6a)$$

A free vibration analysis would proceed with the solution of (9) as before but no assumption as to uniformity would be made. The resulting general solution

$$\psi(x) = AF(x) + BG(x) + CA(x) + DJ(x) \qquad (44)$$

can be obtained for sufficiently well behaved $EI(x)$ and $m(x)$.

Boundary conditions (7) and (8) require that

$$AF''(0) + BG''(0) + CH''(0) + DJ''(0) = 0$$

$$AF''(L) + BG''(L) + CH''(L) + DJ''(L) = 0$$

$$AF'''(0) + BG'''(0) + CH'''(0) + DJ'''(0) = 0$$

$$AF'''(L) + BG'''(L) + CH'''(L) + DJ'''(L) = 0$$

This leads to the characteristic determinant

$$\begin{vmatrix} F''(0) & G''(0) & H''(0) & J''(0) \\ F''(L) & G''(L) & H''(L) & J''(L) \\ F'''(0) & G'''(0) & H'''(0) & J'''(0) \\ F'''(L) & G'''(L) & H'''(L) & J'''(L) \end{vmatrix} = 0$$

This is an equation in ω and its roots are the system natural frequencies. For each root ω_r there correspond specific values of A, B, C, and D and therefore a specific eigenfunction ψ_r. The nature of the four solutions F, G, H, and J may make the solution of the determinant quite complex.

This is indeed an understatement. The general solution (44) will be a combination of transcendental functions and infinite series. Solution of the determinant would be very time consuming and costly. Additionally, inherent in the previous work has been the assumption that shear defection and rotatory inertia are small factors. This is reasonable for the lowest vibration modes but increasingly becomes a source of error as the mode number increases, say for three or more nodes. The shear defection, for example, is about 15% of the bending deflection for the lowest vibration mode, that of two nodes for L/D = 15. [14] For vibration of about four nodes shear deflection becomes greater than bending deflection.

B. Prohl Sequence

A method which allows account to be taken of shear and rotary inertia and can be solved for the eigenvalues and functions is the technique devised by M.A. Prohl in 1945. It was originally produced for calculating the angular velocities at which flexible rotors would resonate. The actual rotor was simulated by a "lumped mass" system in which the rotor mass was concentrated at a number of stations. Station spacing was a matter of convenience though obviously the greater the number of stations the greater the accuracy. The stations were connected by weightless rods whose bending stiffness and shear characteristics were identical to those of the rotor section. This effective discretization allows problem solution by numerical techniques. The procedure is directly applicable to the analysis of ship hull vibration as will be shown. Let

m_i = station mass plus added mass

M_i = bending moment

V_i = shear force

g_i = vertical deflection

θ_i = slope

L_i = length between adjacent stations

ω = angular velocity

KGA_i = shear stiffness which is constant along L_i

EI_i = bending stiffness which is constant along L_i

J_i = mass moment of inertia

and i = subscript denoting consecutive stations.

The equations from strength of materials which govern the motion of the stations and which include shear and rotary inertia terms are

$$\theta_{i+1} = \theta_i + \frac{M_i L_{i+1}}{EI_{i+1}} + \frac{V_{i+1} L_{i+1}^2}{2EI_{i+1}} \tag{1}$$

$$M_{i+1} = M_i + V_{i+1} L_{i+1} - J_{i+1} \omega^2 \theta_{i+1} \tag{2}$$

$$y_{i+1} = y_i + \theta_i L_{i+1} + \frac{M_i L_{i+1}^2}{2EI_{i+1}} + \frac{V_{i+1} L_{i+1}^3}{6EI_{i+1}} - \frac{V_{i+1} L_{i+1}}{KGA_{i+1}} \tag{3}$$

$$V_{i+1} = V_i + m_i \omega^2 y_i \tag{4}$$

It is to be noted that the mass term m_i includes the mass of hull and cargo and the entrained water or "added mass." This will be discussed in Section C.

The relationships between the loading (shear), bending moment and deflection curves are well known from strength of materials. As ω increases, the effect of including the rotary inertia and shear deflection terms in the equations becomes obvious, the shear and moment terms are altered increasingly.

The boundary conditions for this free-free beam are

$$M_S = M_B = 0$$

and

$$V_s = V_B = 0$$

where subscripts s and B represent the stern and bow respectively (more accurately the after terminus and the forward terminus). Thus for station one

$$V_1 = V_s = 0 \tag{5}$$

$$\theta_1 = \theta_s + \frac{M_s L_1}{EI_1} + \frac{V_1 V_1^2}{2EI_1} = \theta_s \tag{6}$$

$$M_1 = M_s + V_1 L_1 - J_1 \omega^2 \theta_1 = -J\omega^2 \theta_s \tag{7}$$

$$y_1 = y_s + \theta_s L_1 + \frac{M_s L_1^2}{2EI_1} + \frac{V_0 L_1^3}{GEI_1} - \frac{V_1}{KGA_1}$$

or

$$y_1 = y_s + \theta_s L_1 \tag{8}$$

It can be seen by inspection of equation (1) through (4) that the values for the bending moment and shear force at any point in the ship will be a function of y_s, θ_s and ω in addition to the calculated ship parameters. If, at the start, y_s is set equal to one, a unit defection of the stern, then the equationsfor the shear and bending moment at the bow will be expressed by the following:

$$V_B = a + b\theta_s$$

$$M_B = -c + d\theta_s$$

where a, b, c and d may be functions of ω. From the boundary condition requirement it is known that both of these equations must equal zero. Arbitrarily setting $M_B = 0$ and $V_B = a + \frac{bc}{d}$ and performing the sequence calculation a number of times for various values of ω a curve of V_B versus ω can be obtained. This curve represents the exciting force V_B which would need to be applied at a specific frequency ω to generate a unit deflection at the stern. The zeros of this curve, that is where the curve crosses the ω-axis, represents the natural frequencies of the vessel where, with the absense of damping, a vanishingly small exciting force would set up synchronous motion. With the natural frequencies obtained the governing equations (1) through (4) will yield the displacement, shear, moment and slope for each station. The displacements for each frequency ω_r are considered for this discrete case to be equivalent to the continuous system eigenfunctions and are called eigenvectors. Indeed as the number of stations becomes large the eigenvector will converge to the eigenfunction. As for the continuous case, the eigenvector must be normalized. A convenient normalization scheme is according to the following

$$\sum_{i=1}^{N} M_i y_i^2 = 1 \qquad (9)$$

which is the discrete representation of (32) in Section A.

C. Added Mass

The preceding technical treatment of the free and forced vibration of a ship like beam involved a mass term which was either uniform or not along the length. This term included not only the mass of ship structure, machinery, cargo, etcetra as might be obtained from a weight curve but also a hydrodynamic "added mass." When a vibrating body is immersed in a fluid (air or water, for example) the fluid surrounding the body is put into continual motion which requires the expenditure of energy. Under conditions of synchronous or resonant motion, the body with this entrained water vibrates. This additional mass serves to lower significantly the natural frequencies of the vessel. It is therefore critical that a determination of this mass be made and added to that of the vessel. The total mass then, called "virtual" mass is the sum of the ship mass and the added mass. This phenomenon has been observed and treated for a few objects in classical hydrodynamics. In the case of an infinitely long circular cylinder having a cross-section of radius r translating perpendicularly to its axis with velocity V in an infinite inviscid fluid it is possible to derive the

potential flow and to calculate the kinetic energy in the fluid. It is found to be [20]

$$T = \frac{1}{2}\rho\pi r^2 V^2$$

per unit length of cylinder where ρ is the fluid density. Letting $M' = \rho\pi r^2$ it is apparent that M' is the mass of fluid that is displaced by unit length of the cylinder. It is also apparent that the effect on the cylinder motion from the presence of the fluid can be allowed for by an addition of M' to the mass per unit length of the cylinder. An energy balance equation for forced motion becomes

$$\frac{d}{dT}\left(\frac{1}{2}MV^2 + \frac{1}{2}M'V^2\right) = FV$$

where F is the external force.

On simplifying

$$(M+M')\frac{dV}{dt} = F$$

or

$$\frac{Mdv}{dt} = F - M'\frac{dV}{dt}$$

Here the presence of the fluid is equivalent to a force per unit length opposing motion. Obviously this virtual mass effect is only present in the case of accelerated motions such as vibration. For a sphere of radius r the kinetic energy of the fluid has been shown to be [20]

$$T = \frac{1}{2}\frac{2}{3}\pi\rho \quad r^3V^2$$

Here $M' = \frac{2}{3}\rho\pi r^3$ which is one half of the mass of displaced fluid. In general

$$M' = C \text{ times the mass of displaced fluid}$$

where C is a vertical added mass coefficient. Comparison of the added mass coefficients for the cylinder and sphere both of radius r shows the former to be twice the latter. This reveals the three dimensional effect. The general method for determining the added mass coefficients of a ship hull is a stepped procedure utilizing first a two dimensional analysis then multiplying by a three dimensional correction called a "J" factor which will be discussed later. The discrete analysis of the Prohl sequence is inherently compatible with this. At each station the geometric and material characteristics are assumed to be constant along the station length. Thus the station can be considered to

be a portion of an infinitely long cylinder and the added mass can be determined. The cylinder is supposed to consist of the underwater hull cross section and its above water mirror image. The actual ship added mass is thus one-half of this computed amount. This analysis is based on the fact that the fluid motion is streamline throughout, that is, irrotational. This is reasonable since the amplitudes and absolute velocities are small.

This assumption of irrotational flow implies that the curl of the fluid particle velocity vector field is zero, that is,

$$\nabla \times \overline{V} = 0$$

where ∇ is the "del" operator. This implies that the velocity vector field is the gradient of a potential function

$$\overline{V} = \nabla\phi \tag{1}$$

Additionally where the flow is incompressible and there are no sources or sinks a fluid continuity equation can be written in the form

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \tag{2}$$

or upon substitution of (1) into (2)

$$\nabla^2\phi = 0 \tag{3}$$

This is the well known Laplace's Equation. The kinetic energy of any flow where a potential exists is [20]

$$2T = \int \rho(\nabla\phi)^2 dV$$

where dV is a volume element and ρ is the fluid density. As has been shown the added mass can be determined from the kinetic energy of the flow. It remains then to compute the potential for the flow around a ship section.

Classical hydrodynamics has evaluated the potential for a very few geometric shapes. In two dimensions flows past circles and ellipses have closed form equations and spheres and ellipsoids of revolution have equations for the potential in three dimensions. A method which greatly expands the number of shapes which can be analyzed is called "conformal mapping" which involves application of analytic function theory and functions of a complex variable.

To illustrate, let

$$w = f(z) \tag{4}$$

where

$$w = u + iv$$

$$z = x + iy$$

(4) then represents a mapping of all points in the z plane onto the w plane. If the function f(z) is single valued then the mapping is one to one, that is to each point z where f(z) is defined there exists one and only one value of w = u + iy in the w plane. The inverse will also be true if f(z) is analytic and $f'(z) \neq 0$. If these conditions are met, then it is also true that relative angle and shape are preserved in such a mapping (thus the term name "conformal"). Right angles in the z plane will be mapped to right angles in the w plane. Another property of complex analytic functions is that the real and imaginary parts satisfy Laplace's equation and therefore can represent fluid flows. Knowing that the potential and stream functions satisfy the Cauchy-Riemann equations

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

and Laplace's equation (3) an analytic complex function can be generated by setting the potential equal to the real part and the stream function equal to the imaginary part

$$\Phi(2) = \phi(x,y) + i\psi(x,y)$$

This function is often called the complex potential. Now returning to (4) and writing its inverse

$$z = F(w)$$

it is apparent that the complex potential $\Phi(z)$ is expressible as a function of w in the form $\Phi[f(w)]$

Further this function is analytic and therefore represents a new fluid flow in the w plane. Each of the equipotential lines, boundary lines and streamlines in the original plane are mapped into the w plane. The orthogonal relationship is present and thus the actual flow around the transformed boundary is known. For example, the flow past a two dimensional body of arbitary shape can be determined (theoretically) by generating a transformation function $w = f(z)$ which will map the points of the boundary of a unit circle in the z plane onto the boundary of the arbitary shape in the w plane. The inverse of the function is then substituted into the complex

potential for the flow in the z plane which is well known. Thus a complete solution for the flow about the arbitary shape is obtained. In practice the inverse may not be manageable or only a rough approximation to the desired shape can be transformed from the unit circle. Cross-sections of ships can, with good accuracy, be transformed from a unit circle however.

The first and definitive work in this area was presented in 1929 by Professor Frank M. Lewis in the Transactions of the Society of Naval Architects and Marine Engineers. The language of the paper is a bit quaint and some of the questions raised have long since been answered, but the method presented for computation of added mass has seen only very minor modifications to the present day.

Let x,y be the coordinates of any point in a two dimensional flow and ϕ,ψ the values for the velocity potential and stream function at the same point. The equation

$$z = x + iy = f(\phi + i\psi)$$

where f designates an arbitrary function represents a two dimensional irrotational flow. The flow in the x, y plane can be further transformed to any other plane X, Y by the relation

$$Z = X + iY = f_1(x + iy)$$

where f_1 is another arbitratry function. Z is of course still a function of $\phi + i\psi$.

Take the flow past a unit circle

$$x^2 + y^2 = 1$$

or in polar form

$$r=1$$

and transform it by means of the relation

$$Z = X+iY = Z+\frac{a}{z}+\frac{b}{z^3} = z+az^{-1}+bz^{-3} \tag{5}$$

Now

$$z = x + iy = re^{i\theta} = e^{i\theta} \tag{6}$$

so

$$X + iY = e^{i\theta} + ae^{-i\theta} + be^{-ei\theta} \tag{7}$$

Since

$$e^{i\theta} = \cos\theta + i\sin\theta$$

it follows on equating real and imaginary parts that

$$X = (1+a)\cos\theta + b\cos 3\theta$$

$$Y = (1-a)\sin\theta - b\sin 3\theta \qquad (8)$$

These are the parametric equations of the transformed circle. The semi-axis parallel to the flow ($\theta=0$) will be of length

$$X_1 = 1 + a + b$$

and the semi-axis perpendicular to flow ($\theta=\pi/2$) is

$$Y_1 = 1 - a + b$$

so the ratio of half beam over draft will be

$$H = B/D = \frac{1-a+b}{1+a+b} \qquad (9)$$

For the flow past a unit circular cylinder ϕ and ψ are given by

$$\phi = U(r + \frac{1}{r})\cos\theta \qquad \psi = U(r - \frac{1}{r})\sin\theta$$

and on the boundary n=1

$$\phi = 2U\cos\theta \qquad \psi = 0 \tag{10}$$

This represents the flow at the boundary of a stationary cylinder and, also represents the flow at the boundary of the transformed cylinder. To obtain the solution for a moving cylinder in a fluid stationary at infinity a superposition of a uniform flow of -U must be made. For this uniform flow

$$\phi = -UX \text{ and } \psi = -UY \tag{11}$$

so using (11) and (8) in (10)

$$\phi = U[(1-a)\cos\theta - b\cos 3\theta] \tag{12}$$

$$\psi = -U[(1-a)\sin\theta - b\sin 3\theta] \tag{13}$$

The kinetic energy of the fluid is

$$2T = \rho\int\int(V_x^2 + V_y^2)\,dxdy$$

and since

$$V_x = \frac{\partial\phi}{\partial x} \qquad V_y = \frac{\partial\phi}{\partial y}$$

$$2T = \rho\int\int[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2]\,dxdy$$

An application of Green's theorem allows the double integral to be replaced by a line integral computed on a path which encloses the area. In this case the fluid is bounded by the cylinder surface and by a circle whose radius is allowed to approach infinity.

$$2T = \rho\iint[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2]dxdy = -\rho\int_{C_1+C_2} \phi\frac{\partial\phi}{\partial n}\,ds \tag{14}$$

where ds is an element of path length, ∂n is an inwardly pointing element normal to the curve, C_1 is the cylinder circumference and C_2 represents the circle with radius approaching infinity. Since the fluid is at rest far from the cylinder the line integral over C_2 will be zero. Since

$$\frac{\partial\phi}{\partial n} = -\frac{\partial\phi}{\partial s}$$

the kinetic energy is given by

$$2T = \rho\int_{C_1}\phi\partial\psi \tag{15}$$

which considering that the expressions for ϕ and ψ are dependent only on θ equals

$$2T = \rho\int_{0}^{\theta=2\pi} U[(1-a)\cos\theta-b\cos3\theta][-U[(1-a)\cos\theta-3b\cos3\theta]]d\theta$$

$$= \pi U^2\rho[(1-a)^2+3b^2] \tag{16}$$

If the semi-axis of the transformed circle is to be B instead of (1-a+b) this energy becomes

$$2T = \pi U^2 \rho [(1-a)^2+3b^2] \frac{B^2}{(1-a+b)^2}$$

$$= \pi U^2 \rho B^2 [\frac{(1-a)^2+3b^2}{(1-a+b)^2}] \qquad (17)$$

As previously mentioned the kinetic energy of flow past a circular cylinder of radius B is

$$2T = \pi U^2 \rho B^2 = M'U^2$$

therefore the virtual mass coefficient for the transformed circle is

$$C = \frac{(1-a^2)+3b^2}{(1-a+b)^2} \qquad (18)$$

and (17) becomes

$$2T = C\pi B^2 \rho U^2 = M'U^2$$

and the added mass per unit length is

$$M' = C\pi B^2 \rho$$

For the ship hull floating on the surface this is reduced by one-half so

$$M' = \frac{1}{2}C\pi B^2\rho \tag{19}$$

Solving (18) and (9) simultaneously for a and b yields

$$b = H\left[\frac{-(2CH-H+1)+(H+1)\sqrt{4C-3}}{2(CH^2-H^2-H-1)}\right] \tag{20}$$

$$a = \frac{(1-H)(1+b)}{(1+H)} \tag{21}$$

and the parametric equation of the transformed circles when of draft D instead of (1+a+b) are

$$X = D\left[\frac{(1+a)\cos\theta+b\cos 3\theta}{(1+a+b)}\right] \tag{22}$$

$$Y = D\left[\frac{(1-a)\sin\theta-b\sin 3\theta}{(1+a+b)}\right] \tag{23}$$

X in this case being vertical and the line $\theta=0$ coinciding with the X axis. In [22] is a collection of graphs for various values of H(half beam to draft ratio) and C. The H values correspond to common ship cross sections. For each of the paired values of C and H the constants a and b were computed from (21) and (20) then the shapes of the transformed circle were plotted from (22) and (23) for values of θ from 0 to $\pi/2$. The added mass for any ship section is then

computed as follows: a visual comparison of the shapes in [22] is made with the body plan for the station. Of course, the body plan must be properly dimensioned and the value of H known. The C value for the actual ship section is obtained by interpolation between adjacent curves and the added mass per foot for the ship section is obtained from (15) remembering that B is half beam. This procedure is the basis for present day calculations. Some refinements which make the procedure programmable are presented in a paper by L. Landweber and M. Macagno in 1967 in the Journal of Ship Research. An equation of the form

$$z = w + \frac{A_1}{w} + \frac{A_2}{w^3} + \frac{A_3}{w^5} \ . \ . \ . \qquad (24)$$

(of which (1) is a truncated version) can describe either the mapping of a hull shape to a circle or a circle to a hull shape. The constants A_i will of course be different. Furthermore, inversion formulas exist which enable the coefficient of one transformation to be determined if the inverse coefficients are known. Let (24) represent the mapping of a wetted hull-shaped contour consisting of a wetted hull and its mirror image across the waterline from the z plane into the w plane. S is the bounded area of the hull contour and r_o is the radius of the circle in the w plane. An expression for the added mass coefficient C_v is [21]

$$C_v = \frac{1}{B^2}[2(r_o^2+A_1)-\frac{S}{\pi}] \tag{25}$$

Let the inverse of (20) be of the form

$$T = z + \frac{B_1}{z} + \frac{B_2}{z^3} + \frac{B_3}{z^5} \ldots \tag{26}$$

A method of determining the coefficients B_i is based on the property that, among the closed curves obtained from (22) by varying the coefficients, the curve which will bound the maximum area will be a circle. Let the ship curve be termed C_T and its mapping in the t plane be termed C_1. If $z = r(\theta)e^{i\theta}$ then

$$\oint_{C_T} \bar{z}\, dz = \oint_{C_T} r(dr+ird\theta) = i \oint_{C_T} r^2 d\theta = ZiS$$

where $\bar{z}$ is the complex conjagate of z. Simlarly for C_1

$$\oint_{C_1} dt = ZiS_1 \tag{27}$$

where S_1 is the area bounded by C_1. Also

$$\bar{T} = \bar{z} + \frac{B_1}{\bar{z}} + \frac{B_2}{\bar{z}^3} + \frac{B_3}{\bar{z}^5} \ldots \tag{28}$$

and

$$dT = (1 - \frac{B_1}{z^2} - \frac{3B_2}{z^4} \ldots)dz \tag{29}$$

After substitution of (28) and (29) into (27) the condition that S_1 be a maximum that is

$$\frac{\partial S_1}{\partial B_i} = 0 \qquad i = 1,2,\ldots n$$

yields a set of n linear equations for the coefficients B_i, i = 1 to n. Standard matrix manipulation techniques are available for then determining the B_i. The difficulties with this method are that the convergence rate with respect to n is relatively low and the accuracy of some intermediate terms decays with increasing n. The solution to this problem is to cut off the first mapping at n=8 and then map the nearly circular curve C_1 from the t plane into a much more nearly circular curve C_2 in the w plane where

$$w = t + \frac{C_1}{t} + \frac{C_2}{t^3} + \frac{C_3}{t^5} \ldots$$

The C_i are determined in much the same way as were the B_i. The coefficients A_i of equation (24) can then be determined from the intermediate coefficients B_i and C_i. The added mass coefficient C_v is then determined from (25).

This thesis makes use of the above outlined procedure to compute the added mass for vertical vibration of hull sections as defined by offset data for each section.

Another significant item in this treatment is to calculate the three dimensional effect mentioned briefly earlier. Lewis, in his paper, talks of a "J" factor where

$$J = \frac{\text{Actual Kinetic Energy of Fluid}}{\text{Kinetic energy of fluid if motion is confined to transverse planes}}$$

J obviously depends on the underwater configuration and will approach unity as the ratio of length to beam gets larger and larger. That is to say as variations of the hull from section to section diminishes.

An ellipsoid of revolution is one of the very few shapes for which an exact solution has been obtained for the three dimensional flow. It is this shape that is used for the computation of the J factor for a vibrating ship. As the vibration mode goes up, and the number of nodes increases, it is apparent that the movement of water becomes more longitudinal and less transverse so that J_i values, where i represents the ith mode, will decrease. The calculation of J_2 and J_3 was made in the paper by Lewis. The treatment is similar to that presented for the two dimensional case in that an equation like (1) for the kinetic energy of the fluid i used

$$2T = -\rho \int\int \phi \frac{\partial \phi}{\partial n} \, ds \qquad (31)$$

where in this case ds is an element of area. The solution of this equation can be obtained due to a closed form solution to Laplace's equation having been found for this flow. That is, the potential ϕ for vibration of two and three nodes can be substituted into (31) and values for the actual three dimensional kinetic energy can be calculated. The denominator in the expression for J (30) is computed by summing (integrating) the values obtained from a two dimensional analysis of cross sections of the body. Lewis tabled the values of J_2 and J_3 for a variety of ratios of L/B. The values are shown in Table 1.

The analysis performed by Lewis required that the vibrating cross section remain in the same plane, that is, the motion results from pure shear. This does not reflect reality and consequently there has been much discussion on the subject beginning with a written comment of Lewis' paper by his peer J. Lockwood Taylor [22][30] and continuing to the present day.

Current procedure [32] involves the determination of J by accurate measurement of the natural vibration frequencies of a given body in air and in water. For Lewis' circular ellipsoid of revolution a simple analysis is possible because the distribution of added mass is directly proportional to the distribution of actual mass (C_v = constant along the entire length). The mass distribution is

$$M_B = \pi r^2 \rho_B \tag{32}$$

where r is the radius of the circular cross section and a function of longitudinal position, ρ is density and subscript B refers to the body. The corresponding added mass is

$$M' = C_v J \pi r^2 \rho_w \tag{33}$$

where C_v=1 and subscript w refers to the water in which the body is immersed. J here is considered to be a constant value applied at each cross section so that the actual three dimensional added mass is obtained from the two dimensional analysis performed. From basic vibration theory [22]

$$\omega_n = \sqrt{K/M}$$

where K represents stiffness and M mass. Put another way

$$\left(\frac{f_a}{f_w}\right)^2 = \frac{M_w}{M_a} \tag{34}$$

where f represents frequency and subscript a represents air. This equation for the circular ellipsoid considered is

TABLE 1

L/B	LEWIS J_2	TOWNSIN J_2	LEWIS J_3	TOWNSIN J_3
∞	1.000		1.000	
30		.947		.924
22.4	.954		.932	
16		.883		.842
15.82	.922		.890	
12.95	.897		.859	
12		.837		.793
10.02	.859		.808	
9		.764		.696
8.48	.829		.770	
7.11	.789		.725	
7		.683		.591
5.83	.741		.670	
5.07	.704		.629	
5		.593		

$$\left(\frac{f_a}{f_w}\right)^2 = \frac{\rho_B \int \pi r^2 dL + J\rho_w \int \pi r^2 dL}{\rho_B \int \pi r^2 dL}$$

$$= 1 + \frac{J\rho_w}{\rho_B} \qquad (35)$$

where the vibrating mass in air M_a is assumed to be due only to the mass of the body and dL is differential length. Equation (35) is reduced to

$$J = \left[\left(\frac{f_a}{f_w}\right)^2 - 1\right] \frac{\rho_B}{\rho_w} \qquad (36)$$

Values from this equation obtained by R.L. Townsin [32] are listed in Table 1 and a comparison with the Lewis values shows a significant decrease reflecting the fact that no restrictions or assumptions were made concerning the cause of the motion. For ship-like vibrating bodies the procedure is somewhat more complex. Frequencies in water and air are both measured experimentally and calculated. For the water calculation several assumed values of J are used to obtain a plot of J_n versus f. A reduction constant obtained from comparison of the two air frequencies is used to correct the measured water frequency. This abscissa is used to obtain the J_n ordinate from the J_n versus f plot. Townsin proposes and this thesis employs the following empirical formula for J_n as a function of B/L

$$J_n = 1.02 - 3(1.2-1/N)B/L \qquad n=2,7 \tag{37}$$

This formula agrees closely with data through J_7.

The remaining consideration which is presented deals with a "sectional longitudinal inertia correction" labelled, by the author, j_i (the lower case to reflect a relation with the Lewis J factor). In most determinations of actual added mass per section an equation of the form of (19) is consistently used. That is

$$m_i' = \frac{1}{2}C_i \pi B_i^2 \rho \tag{38}$$

and

$$\frac{M'}{\Sigma m_i'} = J$$

or

$$M' = J \sum_{i=1}^{N} m_i \tag{39}$$

so

$$M_i' = \frac{1}{2}C_i B_i^2 \rho J \tag{40}$$

where m_i represents the 2-D section added mass, C_i and B_i are the section added mass coefficient and half beam respectively, M' is the 3-D added mass for the entire body and M_i' is the supposed 3-D section added mass. The departure from reality in (40) is apparent. J of the entire body represents a fixed fraction which when multiplied by the added mass of a 2-D flow gives the actual 3-D added mass for the entire body. There are some sections of a vibrating ship where the fluid motion is in fact very close to two dimensional and the added mass should be very close to that calculated from (38). At other sections, particularly near bow and stern, the flaw departs quite significantly from a pure two dimensional flow and the added mass would be less than that obtained from (40). To take into account these aspects the author proposes to use j_i.

Let the added mass at section i for a 2-D analysis be

$$m_i' = \frac{1}{2} C_i \pi B_i^2 \rho \tag{38}$$

and let the actual 3-D added mass be

$$M_i'' = \frac{1}{2} C_i \pi B_i^2 \rho j_i \tag{41}$$

then

$$M' = \sum_{i=1}^{N} M_i'' = \sum_{i=1}^{N} m_i j_i \tag{42}$$

The requirement that equations (39) and (42) yield the same value for M' requires that

$$J \sum_{i=1}^{N} m_i' = \sum_{i=1}^{N} m_i j_i \tag{43}$$

Let

$$j_i = 1 - d_i b \tag{44}$$

where d_i is a difference factor computed for each section by

$$d_i = \frac{|(m_{i+1}' - m_i') + (m_i' - m_{i-1})|}{2} + \frac{n \sum_{i=1}^{N} m_i'}{2N}$$

$$= \frac{|m_{i+1} - m_{i-1}|}{2} + \frac{n \sum_{i=1}^{N} m_i'}{2N} \tag{45}$$

and b is a normalization constant to be determined by substitution of (44) into (43). In (45) N is the number of stations and n is the number of nodes in the vibrating mode. As the number of nodes increase three dimensional flow increases at all stations so $j_i \to J$. An example will serve to illustrate. Assume that an analysis has determined the 2-D sectional

added masses to be as listed in column 2 of Table 2. The difference factors from (45) are listed in column 3. Substituting the totals of columns 2 and 4 into (43) yield

$$.81(692) = 692 - (28380 + 47886n)b$$

$$b = 131.5/(28380 + 47886n)$$

$b(x10^{-3})$	n
4.633	0
1.723	1
1.059	2
0.764	3

Column 5 lists M_i' or the $m_i'J$. Columns 6, 7 and 8 show the actual added mass at each station for vibrations of 0, 1 and 3 nodes respectively. Notice as n increases the added mass approaches the values in column 5.

TABLE 2

1	2	3	4	5	6	7	8
	m_i'	d_i	$j_i \times m_i'$	M_i'	M_i''		
	0				n=0	n=1	n=3
1	85	70. + 69.2n	85-(5950+5882n)b	68.9	57.5	64.7	67.0
2	140	43.5+ 69.2n	140-(6090+9688n)b	113.4	111.7	112.9	113.1
3	172	12.5+ 69.2n	172-(2150+11902n)b	139.3	162.0	147.8	143.1
4	165	21. + 69.2n	165-(3465+11418n)b	133.7	149.0	139.4	136.2
5	130	82.5+ 69.2n	130-(1072+8996n)b	105.3	80.3	96.1	101.2
	0						
	Σ = 692		Σ=692-(28380+47886n)b	Σ=560.6	560.5	560.9	560.6

D. Wave Excitation

The forcing function $f(x_i,t)$ used in the previous treatment represents the total external force per unit length at any section x_i and any time t. It includes the many effects of the waves encountered, the motion of the ship and the fluid flow. It is, as such, very complicated and the subject of much research. [12][16]

The most simple formulation of wave excitation would be due strictly to buoyancy changes from wave passage.

$$f(x_i,t) = \int_{V_o}^{V} \gamma \, dV$$

where an elemental volume dV is equal to $b_i dy$ to give

$$f(x_i,t) = \int_{0}^{\zeta(t)} \gamma b_i dy \qquad (1)$$

where b_i is the breadth of the ship and a function of y, ζ is the wave height at any time t and γ is the specific weight of water. It has been traditional in naval architecture to use a trochoidal wave [14] to model the wave form for deep water waves. With sharper crests and flatter troughs it makes a very close approximation to the actual sea surface. However, this wave form cannot be derived from the velocity potential for surface waves and the parametric equations which describe a trochoid are an additional complexity in any analysis.

A wave form which is more manageable and is derived from hydrodynamic theory of waves of finite amplitude was prepared by G.G. Stokes in 1847 [14].

$$\zeta = \bar{\zeta}\cos(\omega t - Kx) + \frac{K\bar{\zeta}^2}{2}\cos 2(\omega t - Kx) \tag{2}$$

where $\bar{\zeta}$ is half wave height from trough to crest and the circular frequency ω and the wave number K are related by

$$K = \omega^2/g = 2\pi/L_w = g/V_w^2 \tag{3}$$

for deep water waves where L_w is the wave length and V_w is the celerity or wave velocity. In (2) a simple traveling cosine wave is modified by a harmonic which is half the length of the fundamental but which moves with the same celerity. This wave form has the same sharp peaks and wider troughs as does the trochoid but the fact that it is nonlinear with respect to height makes it somewhat less attractive. In the program suitable approximations will be made.

It is well known that the pressure in a still body of water is equal to γh where h is the distance below the still surface. It is also well known that in deep water the water particles have a circular motion which serves to reduce the pressure according to Bernoulli's theorem. That is, for a simple harmonic wave,

$$p = \gamma h + \bar{\zeta}\gamma e^{-Kh}\cos(\omega t - Kx) \tag{4}$$

This is the Smith effect. The pressure on the hull due to the wave passage is then

$$p_m = \zeta\gamma e^{-Kh} \tag{5}$$

where ζ is given by (2). The buoyancy due to this pressure is an integration of the vertical component of the pressure over the surface area of a hull slice of unit length. That is

$$f(x,t) = -\int_0^s p_w \, \bar{n}\cdot\bar{j} \, ds$$

where $\bar{n}$ is an outward pointing unit normal to the hull and $\bar{j}$ is the unit vector in the positive y direction. For a ship vertically sided to the baseline this reduces to

$$f(x,t) = p_w(T)b$$

or with (5)

$$f(x,t) = b\zeta\gamma e^{-KT} \tag{6}$$

where T is the draft.

Since the pressure given by (5) is seen to, in effect, reduce the waveheight ζ, an effective wave height ζ^* can be defined so that

$$f_1(x,t) = \gamma b \zeta^* \tag{7}$$

where the assumption is made that the ship is vertically sided at the waterline so that b is constant over the range of ζ^*. The subscript 1 indicates that this buoyancy force is only part of the total excitation. ζ^* is given by [16]

$$\zeta^* = \zeta[1-\frac{K}{b}\int_{-T}^{0} ze^{Ky}dy] \tag{8}$$

where z is the hull offset and y is distance to the design waterline. The integral serves to weigh the determination of the reduction factor by the hull shape. For a section with a vertical side to the baseline (8) would reduce to

$$\zeta^* = \zeta[1-K\int_{-T}^{0} e^{Ky}dy]$$

$$= \zeta[1-\int_{-T}^{0} e^{Ky}Kdy]$$

$$= \zeta[1-(1-e^{-KT})]$$

$$= \zeta e^{-KT}$$

so that

$$f_1(x,t) = \gamma b \zeta e^{-KT}$$

which is identical to (6). The discrete form of (8) is

$$\zeta^* = \zeta[1-\frac{K}{b}\sum_{i=1}^{n} z_i\ e^{Ky_i}\ \Delta y_i] \tag{9}$$

and for the discrete forcing function per unit length

$$f_1(x_j,t) = \gamma b_j \zeta^* = \gamma b_j\ \zeta[1-\frac{K}{b_j}\sum_{i=1}^{n} z_i\ e^{Ky_i}\ \Delta y_i] \tag{10}$$

where ζ is given by (2).

The behavior of the bracketed term is to reduce the wave height as K increases. For small K the term is very close to 1. Consider the straight sided vessel again. A reduction in wave height to one-tenth of the long wave value requires

$$.1 = e^{-KT}$$

$$-2.30 = -KT$$

$$K = T/2.30$$

For a draft of 20 feet K = 8.70 which corresponds to a wave length of .72 feet. So as K increases the correction term approaches zero. Thus there are two major effects taking place to reduce the energy available for excitation of the higher modes of a ship hull. First the wave height decreases as the frequency increases and the effect on an immersed body decreases as the frequency increases.

Due to the logarithmic decrease in (9) the discrete form of the correction term, hereinafter called D, causes some convergence problems, that is, as K increases D fails to approach zero if the increment Δy_i is too large. In the program four foot waterlines are used and the following slightly modified expressions must be employed.

$$D = e^{-KTS}$$

$$S = \frac{1}{b_i n} \sum_{i=1}^{n} Z_i \qquad (11)$$

Equation (11) has the proper limiting values and accounts for the hull shape also.

There are two other parts to the forcing function to be presented. As the wave passes down the side of the ship the vertical component of the orbital velocity has a damping effect and this force can be expressed by

$$f_z(x_j,t) = (N'-Vdm'/dx)\dot{\zeta}^* \qquad (12)$$

where (N'-Vdm'/dx) is the sectional hydrodynamic damping coefficient which will be discussed later and $\dot{\zeta}^*$ is the time derivative of the effective wave height. The third force is due to the inertia of the added mass

$$f_3(x_j,t) = m'_j \ddot{\zeta}^* \tag{13}$$

where $\ddot{\zeta}$ is the effective vertical acceleration of the wave height. As was discussed in Section C the added mass may be incorporated into the equation of motion in two ways, either as an addition to the mass of the vessel and placed on the left hand side or as an exciting term and placed on the right hand side. The superposition principle allows the separation of the added mass effect in this case since two separate motions are being superposed, the vibratory motion of the vessel and the harmonic motion of the waves.

In (13) m'_j is the added mass calculated for immersion to the still water waterline. As the wave passes, the draft at a section changes and so does the added mass, but here the sides at the waterline are assumed to be nearly vertical and the change in added mass is very small so the effect can be neglected.

The total excitation force is then

$$f(x,t) = \sum_{i=1}^{3} f_i(x,t) \tag{14}$$

and the modal excitation force is obtained by multiplying by Y_r and summing over the ship length as shown in Section A.

$$F_r(t) = \sum_{i=1}^{N} Y_r(x_i) f(x_i,t) \tag{15}$$

The correct surface height, velocity and acceleration are

$$\zeta^* = D\bar{\zeta}\cos(\omega t-Kx)+\frac{DK\bar{\zeta}^2}{2}\cos 2(\omega t-Kx)$$

$$\dot{\zeta}^* = -D\omega\bar{\zeta}\sin(\omega t-Kx)-DK\omega\bar{\zeta}^2\sin 2(\omega t-Kx) \tag{16}$$

$$\ddot{\zeta}^* = -D\omega^2\bar{\zeta}\cos(\omega t-Kx)-2DK\omega^2\bar{\zeta}^2\cos 2(\omega t-Kx)$$

and

$$f(x_i,t) = \gamma b_i\zeta^*+(N'-Vdm'/dx)\dot{\zeta}^* + m_i\ddot{\zeta}^*$$

$$=(\gamma b_i-m_i'D_i\omega^2\bar{\zeta})\cos(\omega t-Kx)$$

$$-D_i\omega\bar{\zeta}(N'-Vdm'/dx)\sin(\omega t-Kx)$$

$$+\left(\frac{\gamma b_i D_i K\bar{\zeta}^2}{2} - 2D_i K\omega^2\bar{\zeta}^2 m_i\right)\cos 2(\omega t-Kx)$$

$$-D_i K\omega\bar{\zeta}^2(N'-Vdm'/dx)\sin 2(\omega t-Kx) \tag{17}$$

Let

$$G_{i1} = \gamma b_i D_i \bar{\zeta} - m_i D_i \omega^2 \bar{\zeta}$$

$$G_{i2} = -D_i \omega \bar{\zeta} (N' - Vdm'/dx)$$

$$G_{i3} = \frac{\gamma b_i D_i K \bar{\zeta}^2}{2} - 2 m_i K D_i \omega^2 \bar{\zeta}^2 \qquad (18)$$

$$G_{i4} = -D_i K \omega \bar{\zeta}^2 (N' - Vdm'/dx)$$

then

$$\begin{aligned} f(x_i,t) = {} & G_{i1}\cos\omega t\cos Kx + G_{i1}\sin\omega t\sin Kx \\ & + G_{i2}\sin\omega t\cos Kx - G_{i2}\cos\omega t\sin Kx \\ & + G_{i3}\cos 2\omega t\cos 2Kx + G_{i3}\sin 2\omega T\sin 2Kx \\ & + G_{i4}\sin 2\omega t\cos 2Kx - G_{i4}\cos 2\omega T\sin 2Kx \end{aligned} \qquad (19)$$

Substitution into (15) yields

$$\begin{aligned} F_r(t) = {} & \cos\omega t \sum_{i=1}^{N} Y_r(x_i)\, G_{i1}\cos Kx_i A \\ & + \sin\omega t \sum_{i=1}^{N} Y_r(x_i) G_{i1}\sin Kx_i B \end{aligned}$$

$$+ \sin\omega t \sum Y_r(x_i)\, G_{i2} \cos Kx_i C$$

$$- \cos\omega t \sum Y_r(x_i)\, G_{i2} \sin Kx_i D$$

$$+ \cos 2\omega t \sum Y_r(x_i)\, G_{i3} \cos 2Kx_i$$

$$+ \sin 2\omega t \sum Y_r(x_i)\, G_{i3} \sin 2Kx_i$$

$$+ \sin 2\omega t \sum Y_r(x_i)\, G_{i4} \cos 2Kx_i$$

$$- \cos 2\omega t \sum Y_r(x_i)\, G_{i4} \sin 2Kx_i \qquad (20)$$

Or

$$F_r(t) = P_{r1} \cos(\omega t - \theta_1)$$

$$+ P_{r2} \cos(\omega t - \theta_2)$$

$$+ P_{r3} \cos(2\omega t - \theta_3)$$

$$+ P_{r4} \cos(2\omega t - \theta_4) \qquad (21)$$

where

$$P_{r1} = \{(\sum_{i=1}^{N} Y_r(x_i)G_{i1}\cos Kx_i)^2+(\sum_{i=1}^{N} Y_r(x_i)G_{i1}\sin Kx_i)^2\}^{1/2}$$

$$P_{r2} = \{(\sum_{i=1}^{N} Y_r(x_i)G_{i2}\cos Kx_i)^2+(\sum_{i=1}^{N} Y_r(x_i)G_{i2}\sin Kx_i)^2\}^{1/2}$$

$$P_{r3} = \{(\sum_{i=1}^{N} Y_r(x_i)G_{i3}\cos 2Kx_i)^2+(\sum_{i=1}^{N} Y_r(x_i)G_{i3}\sin 2Kx_i)^2\}^{1/2} \quad (22)$$

$$P_{r4} = \{(\sum_{i=1}^{N} Y_r(x_i)G_{i4}\cos 2Kx_i)^2+(\sum_{i=1}^{N} Y_r(x_i)G_{i4}\sin 2Kx_i)^2\}^{1/2}$$

$$\theta_1 = \arctan \frac{\sum_{i=1}^{N} Y_r(x_i)G_{i1}\sin Kx_i}{\sum_{i=1}^{N} Y_r(x_i)G_{i1}\cos Kx_i}$$

$$\theta_2 = \pi\text{-}\arctan \frac{\sum_{i=1}^{N} Y_r(x_i)G_{i2}\cos Kx_i}{\sum_{i=1}^{N} Y_r(x_i)G_{i2}\sin Kx_i}$$

$$\theta_3 = \arctan \frac{\sum_{i=1}^{N} Y_r(x_i)G_{i3}\sin 2Kx_i}{\sum_{i=1}^{N} Y_r(x_i)G_{i3}\cos 2Kx_i} \quad (23)$$

$$\theta_4 = \pi\text{-}\arctan \frac{\sum_{i=1}^{N} Y_r(x_i)G_{i4}\cos 2Kx_i}{\sum_{i=1}^{N} Y_r(x_i)G_{i4}\sin 2Kx_i}$$

The P_{ri} i-1,2,3,4 are known as "participation factors" and were obtained by weighting the wave forces by the mode shape. The factors indicate the participation of the wave forces in exciting a vertain mode. Solution of the independent equations for the modal response as discussed in Section C yields

$$\eta_r(t) = \sum_{i=1}^{4} \frac{P_{ri}\cos(\omega t-\theta_i+\varepsilon_r)}{((\omega_r^2-\omega^2)^2+(C_r\omega)^2)^{1/2}} \tag{24}$$

where

$$\varepsilon_r = \arctan\left(\frac{C_r\omega}{\omega^2-\omega_r^2}\right)$$

The most significant part of this exact time history is the magnitude of the response. Equation (24) may be manipulated to yield the amplitude of the sinusoidal response

$$\eta_r = \{(\sum_{i=1}^{4} P_{ri}\cos\theta_i)^2 + (\sum_{i=1}^{4} P_{ri}\sin\theta_i)^2\}^{1/2} \tag{25}$$

The nonlinearity of the expression for the height of the sea surface can be eliminated by dropping the modifying harmonic. Although the simple cosine term is not an exact representation of the sea surface it is ideal for representing the many component waves of the general sea. It also allows the principle of superposition to be applied and for these reasons the program encorporates this modification. Resultantly $G_3=G_4=P_3=P_4=0$ with significant subsequent simplification of the program.

This analysis assumes that the presence of the vessel does not disturb the wave motion which it, in fact, does. This, then, provides an upper bound to excitation and real forces will be less. How much less is a matter for speculation and research.

E. Damping

It would not be incorrect to say that relatively little is known about damping associated with ship hull vibration. To be able to calculate the amplitudes of desired quantities such as stress levels, bending moments and displacements, the distribution of damping in the hull, cargo and surrounding water should be known. The available literature is sparse, [4][11][16][17] but it is generally assumed that energy is dissipated thru a number of mechanisms. A convenient categorization divides the mechanisms into those which are hydrodynamic and those which are not. Water friction and the generation of both surface and pressure (sound) waves are the hydrodynamic effects. Structural and cargo damping are non-hydrodynamic. Another possible way of differentiating the loss mechanisms is to determine where the damping takes place, either within or external to the hull.

As described previously, superposition of the natural modes of the ship yields for vertical vibration

$$y(x,t) = \sum_{r=0}^{\infty} Y_r(x)\eta_r(t) \qquad (1)$$

where $Y_r(x)$ is the normalized mode shape and $\eta_r(t)$ is the natural coordinate which incorporates the natural frequency, initial conditions and modal damping factor into a time dependent function. Motion for r=0,1 is heave and pitch respectively. Fortunately the damping in ship hull vibration is relatively small so that calculations for the natural modes and frequencies can be carried out assuming no damping without significant error. This is especially true for the lower modes.

Other qualitative information concerning damping has been known for some time. Higher modes are less affected by hydrodynamic damping due to in part the smaller displacements involved. In these modes structural damping is the major factor. The situation is reversed for lower modes. The generation of sound waves increases as the vibration frequency goes up but this remains a very small quantity throughout the range of interest, that is ω less than about 35 rad/sec. For rigid body motion there is, of course, no structural damping. If vibration analysis is of concern and sea-keeping is not then discussion can be limited to distortion modes $r \geq 2$. Surface wave generation increases with decreasing ω. For even the largest of vessels, however, ω_2 is greater than about 2.5 rad/sec and wave making damping is far less than structural damping.[17]

In a discussion of damping it is common to refer to the logarithmic decrement δ associated with a mode or given damping mechanism.

$$\delta = 2\pi\zeta/\sqrt{1-\zeta^2} = \ln(x_1/x_2) \qquad (2)$$

where ζ is the critical damping ratio associated with the mode or mechanism and x_1 and x_2 are values of the response separated by one wave period.

Since structural damping is the significant factor and appears to be manageable there has been some effort directed at its analysis.[4] This damping derives from hysteresis in the steelwork. The logarithmic decrement for this is a power function of the stress. For levels of stress less than about 30,000 psi

$$\delta = C\sigma^n \qquad (3)$$

where C is a material constant and $n \simeq 3$. At higher stress the exponent increases rapidly and other effects such as stress history become important. For steel plate (3) becomes

$$6 \times 10^{-5} \leq \delta \leq 1.6 \times 10^{-4}$$

Values for a welded structure go up with the complexity of the structure and may lie in the range

$$9 \times 10^{-3} \leq \delta \leq 3 \times 10^{-2}$$

This dramatic increase is explained in part by the high residual stresses from welding and stress concentrations at joints. There may also be as yet undiscovered hysteresis effects which are structure related instead of material related.

Ships of current design are usually welded but in riveted ships coulomb friction at the joints is very important. In traditional analysis this dry friction is treated differently from other damping effects. The coulomb damping force is not a function of velocity rather it has a constant value for all velocities and varies only in sign. This directional force is treated as an excitation force which opposes motion and equations for each direction arc solved. One test concluded that riveted ships have twice the structural damping of welded ships.[17] This would obviously vary with ship age and history.

Experiments have been performed on loaded and unloaded vessels to determine cargo damping effects. Results show that cargo does increase the logarithmic decrement but formulations which have been made to quantify the effect have failed to predict well.[17]

The literature separates the generation of surface waves from the other damping mechanisms. It is that part of the hydrodynamic damping that can be derived from potential theory and the only part of ship damping with a solid mathematical base. However, the literature goes on to say that in the distortion modes the natural frequencies are relatively high and the displacements so small that for most existing vessels the effect can be taken as zero. A bit ironic. In equation (11) of Section D the term in parentheses (N'-Vdm'/dx) is the sectional hydrodynamic damping coefficient according to the strip theory approach used in this paper. N' is a parameter which quantifies the generation of surface waves and Vdm'/dx is a forward speed correction term. V is velocity and m' is the added mass. This coefficient, it is seen, varies along the hull length and this distribution of damping effect is important for computation of exciting forces. The normal mode analysis in Section A required that the damping be proportional to the mass and stiffness distribution. Otherwise the modal equations would not be uncoupled. Since the predominant damping source is structural this assumption is not too far off, even though the distribution of the sectional hydrodynamic damping is weighted at the bow and stern. N' is nearly zero amidships but gets larger at the bow and stern. The effect is still very small. Only the forward speed correction will be considered in this work and consequently the amplitude obtained will be slightly high.

Consideration of equation (2) brings up the fact that in much practical usage all the damping effects are lumped into one parameter and its value obtained from a spectral analysis of stress data taken from a vessel which is loaded impulsively so that all modes are excited. Several full scale measurements from slamming or mechanical excitation (anchor drop test) have been obtained and modal damping constants calculated.[11][17] Some convenient empirical relations have been devised from such experiments and other means so that values for the C_r can be obtained quickly. From basic vibration theory [24]

$$C_r = 2\omega_r \delta_r$$

$$= \frac{2\omega_r \delta_r}{\sqrt{(2\pi)^2 + \delta_r^2}} \qquad (4)$$

and a formula for the logarithmic decrement which seems to give intermediate and hopefully representative values is [17]

$$\delta_r = 1.065 \times 10^{-2} \omega_r^{1/2} \qquad \omega < 31.5 \qquad (5)$$

Equations (4) and (5) will be used in this paper to determine the C_r.

3. PROGRAM OVERVIEW

Estimation of the parameters for use in the Prohl sequence calculations is a choice made by the author. The values of m, A, I and J for each section could be obtained from scantling lists and a weight curve but their assembly would be time consuming and defeat one of the purposes of this work, that is rapid vibration information with a minimum of data required as input. Therefore the subroutine HULL for estimating these quantities has been incorporated into the program. Input required is station spacing, number of stations, displacement, thickness of side shell, main deck and bottom plating, offset data and deck edge heights for each station.

Sectional mass is computed by first obtaining the total sectional area from the offset data for each station. A partial sectional area is computed by the use of the five-eight rule [14] and the offset data. An additional area is computed which is equal to the deck edge height minus the highest waterline for which offset data is input times the offset at that waterline. The sectional area equals the sum. A uniform permeability is a reasonable assumption for naval vessels so that the mass is proportional to the area at each station. The sum of the sectional areas is divided into the displacement to get an area density. The density is then multiplied by each sectional area to obtain a

sectional mass. No attempt is made to account for superstructure concentration of mass or its effect on stiffness. A two beam ship model would be the next step in sophistication.

The mass moment of inertia, J, about the neutral axis is determined by first assuming that the beam cross section has a uniform distribution of mass. Naval vessels are characteristicly longitudinally framed. In the midship section the hull strength is provided by the main deck, side shell and bottom plating and their associated longitudinals. A thirty inch spacing is not uncommon and the stiffener area to plate area ratio is about .3. So, the effect of longitudinal is taken into account by increasing the plating thickness by 30%. The neutral axis is determined by a calculation of the first moment of the hull steel. The sectional area density is then used along with the offset data in a second moment calculation about the neutral axis. Each hull cross section is divided by waterlines with four foot spacing. The area from the baseline to the first waterline is assumed to be bounded by a parabola. All subsequent areas are assumed to be trapezoids. The distance from the neutral axis to the centroid of each incremental area is computed, squared and multiplied by the area. The second moment of the area about its own centroid is computed. The sum of all these terms equals the sectional second moment. This is multiplied by the sectional area density to obtain J.

The moment of inertia, I, is obtained by a second moment calculation of the hull steel around the neutral axis.

The effective shear area KA in a hull shape is very difficult to determine. Traditional practice has been to use only the vertical side shell and any continuous longitudinal bulkheads in the calculation. Analytically obtained values for K are for simple symmetric shapes only.[15] Calculations by the author for box type sections with various side, deck and bottom thickness and beam/draft ratios shows that an approximate value for the effective shear area can be obtained from the following

$$(KA)_i = .87(2tD) = 1.74tD \qquad (1)$$

where t is the sideshell thickness and D is the depth. Then for steel

$$KAG_i = 1.74tDE/2(1+\mu)$$

or

$$KAG_i = 2.35 \times 10^8 \ tD \qquad (2)$$

for t in inches and D in feet.

Subroutine NMODES first computes the virtual mass of each station which varies with the mode of vibration for which a solution is sought. Burrill's formula [14] is used to estimate the frequency of two mode vibrations and the J factor for that mode is formed and virtual masses computed. Equations (1) through (4) of Section B are solved for each station given a value of ω in the following way: the shear, bending moment, displacement and slope at any station i are assumed to be of the form

$$V_i = a_i + b_i\theta_s$$

$$M_i = -c_i + d_i\theta_s \qquad (3)$$

$$y_i = e_i + f_i\theta_s$$

$$\theta_i = g_i + h_i\theta_s$$

where θ_s is an unknown initial slope at the stern. Then (1) through (4) of Section B are used to determine the conditions at station i+1

$$a_{i+1} = a_i + m_i\omega^2 e_i$$

$$b_{i+1} = b_i + m_i\omega^2 f_i$$

$$g_{i+1} = g_i - \frac{c_i L}{EI_{i+1}} + \frac{a_{i+1} L^2}{2EI_{i+1}}$$

$$h_{i+1} = h_i + \frac{d_i L}{EI_{i+1}} + \frac{b_{i+1} L^2}{2EI_{i+1}}$$

$$-c_{i+1} = -c + a_{i+1} L - J_{i+1} \omega^2 g_{i+1}$$

$$d_{i+1} = d_i + b_{i+1} L - J_{i+1} \omega^2 h_{i+1} \qquad (4)$$

$$e_{i+1} = e_i + g_i L - \frac{c_i L^2}{2EI_{i+1}} + \frac{a_{i+1} L^3}{6EI_{i+1}} - \frac{a_{i+1} L}{KGA_{i+1}}$$

$$f_{i+1} = f_i + h_i L + \frac{d_i L^2}{2EI_{i+1}} + \frac{b_{i+1} L^3}{6EI_{i+1}} - \frac{b_{i+1} L}{KGA_{i+1}}$$

From the treatment in Section B

$$\theta_s = \frac{c_N}{d_N} \qquad (5)$$

and

$$a_N + \frac{b_N c_N}{d_N} = 0 \qquad (6)$$

at a natural frequency. The subroutine performs the sequence calculation until opposite signs are obtained for the quantity on the left hand side of (6) for two values of ω. A natural frequency is then known to lie between these two values.

The subroutine is designed to determine the natural frequencies to an accuracy of ±.04 radian/sec. After a natural frequency is obtained the mode shape is determined from

$$y_i = e_i + f_i \frac{c_N}{d_N} \tag{7}$$

and normalized by the method discussed in Section B

$$\sum_{i=1}^{N} m_i y_i^2 = 1$$

Data obtained from the Andersson reference [2] for a 60,000 ton tanker was input into NMODES. The resulting natural frequencies obtained are compared with the measured frequencies in radians/sec.

	NMODES	[2]	%
1	4.92	4.89	+ .6
2	10.63	10.43	+1.9
3	16.68	16.41	+1.6
4	23.20	22.91	+1.3
5	29.26	29.17	+ .3
6	34.88	35.03	- .4
7	39.77	40.33	-1.4

The percent difference is quite small.

The input subprogram reads data which is punched according to the format listed after each read statement. The data must be placed in the proper fields, decimal points must be punched and all integers must be right justified. STA is an integer variable and is the number of stations for which offset data is given. STASPA is the station spacing in feet, DISPL is the displacement of the vessel in long tons. The program later changes this value and does all calculations in slugs, BEEM is the beam of the vessel in feet. (Another variable with the correct spelling is used in the program.) T1, T2 and T3 are the main deck, side shell and bottom plating thicknesses respectively of the midship section in inches. DECKE is the deck edge height from bow to stern in feet. There must be one entry per station. DATA is offset data, one card per station. All entries are in decimal feet. The first field is a real integer (decimal point must be included) which is closest to the draft at that station divided by 4. The remaining twelve fields are offset data starting from the bottom and proceeding upward at four foot intervals. The first field must have a real integer less than or equal to twelve and only 12 waterlines may be input, unused fields may be punched with zeros.

The output will list the computed station mass in slugs, the mass moment of inertia J in slug ft^2, the second moment of area I in ft^4, shear stiffness KAG in lbf and the 2-D added mass for each station in slugs. The program will list the first seven natural frequencies and the normalized made shapes. The program will also list the response amplitude operator over the frequency range of interest for speeds of 0, 10, 20 and 30 knots. It will also list the response of the vessel to a Pierson-Moskowitz fully developed sea with a wind speed of 50 knots. Response amplitude is in inches measured at the bow.

It is obvious from a look at the Pierson-Moskowitz formula that precious little energy exists at frequencies above the first mode of vibration, springing. It is reasonable then to assume that wave excitation is significant only for large, long and shallow vessels. The large masses and the low stiffnesses (due to reduced depth) are responsible for a relatively low first natural frequency, one low enough that sufficient energy is available for excitation. Too, as wave frequency goes up the effect on a vessel of constant draft drops off logarithmicly as has been shown. The only times then that the higher modes will be excited are when slamming occurs or under some other impulse type loading. The value of the program lies in the beam parameters obtained

from the HULL subroutine, the natural frequencies and mode shapes obtained from NMODES and the forcing data which can be retrieved from FORCE. The node location in the natural modes is important as possible locations for placement of machinery operating at or near that natural frequency since excitation at a node will not excite vibration.

Numerous comment cards have been inserted in the program and can be used to understand the many small procedures, which are a natural part of any project of this dimension. The text of this thesis serves only to discuss the major technical areas and not the interesting but obviously minor structural details of software construction. The program listing which follows is divided into the subroutines. A sample input record and output record are shown.

```
C       MAIN PROGRAM TO EVALUATE HULL RESPONSE TO WAVE EXCITATION
        COMMON STA,STASPA,BEEM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
       1,STAMAS(40),SKJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
       1A(13,40),RAO(4,50),PERMOS(5,50)
        DIMENSION BEAM(40)
        INTEGER STA
        REAL MODE,KX,LBP,KAPPA,JFACT,KAG
        CALL INPUT
        CALL ADMASS
        CALL HULL
        CALL NMODES
        CALL FORCE
        CALL OUTPUT
        STOP
        END
```

```
      SUBROUTINE INPUT
      COMMON STA,STASPA,BEEM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SRJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40)DAT
     1A(13,40),RAO(4,250),PERMOS(5,250)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
      READ(5,10) STA,STASPA,DISPL,BEEM,DRAFT,DEPTH,T1,T2,T3
   10 FORMAT(I2,8F9.0)
      READ(5,20)(DECKE(I),I=1,STA)
   20 FORMAT(10F8.0)
      D040 J=1,STA
      READ(5,30)(DATA(J,I)I=1,13)
   30 FORMAT(13F6.0)
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ADMASS
      COMMON STA,STASPA,BEEM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SKJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
     1A(13,40),RAO(4,50),PERMOS(5,50)
      DIMENSION Z(91),DZ(90),RS(91),DRO(91),RO(91),P(16),B(8),E(8),TE(91
     1),EI(8,9),TA(91),R(8),CE(8),D(8),G(8),XI(91),ETA(91),TI(91),T(91,1
     16)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
      PP=3.14159265
      DANG=.0174533
      DO 980 LIMA=1,STA
      RS(1)=0.
      Z(1)=0.
      DO 5 J=1,90
      RS(J+1)=0.
    5 Z(J+1)=Z(J)+DANG
      I1=DATA(1,LIMA)
      K=1
    6 CONTINUE
      IF(K.NE.1) GOTO 7
      X1=0.
      X2=0.
      X3=DATA(2,LIMA)
      Y1=4.
      Y2=0.
      Y3=4.
      GOTO 8
    7 X1=X2
      X2=X3
      X3=DATA(K+1,LIMA)
      Y1=Y2
      Y2=Y3
      Y3=Y3+4.
    8 RI1=I1
C     DETERMINE ANGULAR LIMITS
```

```
      IF(K.GE.3) N1=ATAN((4.*RI1-Y1)/X1)/DANG
      M1=1.+ATAN((4.*RI1-Y3)/X3)/DANG
      IF(M1.LT.0) M1=1
      IF(K.LE.2) N1=90
      IF(K-1) 11,680,11
C     DETERMINE HULL BOTTOM PARABOLA
  680 A1=4./X3**2
      B1=0.
      C1=0.
      GOTO 18
   11 CONTINUE
C     DETERMINE PARABOLAS FOR SIDESHELL
      D1=X1**2*X2+X3**2*X1+X2**2*X3-X3**2*X2-X1**2*X3-X2**2*X1
C     D1=0. IMPLIES VERTICAL SIDE OR ZERO INTERCEPT
      IF(D1-.5) 403,600,600
  403 IF(ABS(D1)-1.) 20,20,15
   15 B1=(X1**2*Y2+X3**2*Y1+X2**2*Y3-X3**2*Y2-X1**2*Y3-X2**2*Y1)/D1
   40 A1=(Y1*X2+X1*Y3+X3*Y2-Y3*X2-X3*Y1-Y2*X1)/D1
      C1=(X1**2*X2*Y3+X1*Y2*X3**2+Y1*X2**2*X3-
     1X3**2*X2*Y1-X3*Y2*X1**2-Y3*X2**2*X1)/D1
C     CHECK PARABOLA Y-AXIS INTERCEPT
      IF(C1-4.*RT1) 405,405,640
C     CHECK SLOPE OF THE CURVE
  405 IF(2.*A1*X1+B1) 640,640,18
   20 IF(K.EQ.2) GOTO 640
C     RS(L) FOR A LINEAR SECTION OF PLATING
      DO 30 L=M1,N1
      CZ=COS(Z(L))
      ROOT=X2/CZ
      IF(RS(L).EQ.0.) RS(L)=ROOT
      IF(RS(L).NE.0.) RS(L)=(RS(L)+ROOT)/2.
   30 CONTINUE
      GOTO 100
C     RS(L) FOR A VERTICAL SECTION OF PLATING
  640 M1=1.+ATAN((4.*RI1-Y2)/X2)/DANG
      DO 650 L=M1,N1
```

```
      E1=-TAN(Z(L))
      SLOPE=4./(X2-X1)
      XX=(4.*R11-Y1+SLOPE*X1)/(SLOPE-E1)
      ROOT=SQRT(XX**2+(E1*XX)**2)
      IF(RS(L).EQ.0.)  RS(L)=ROOT
      IF(RS(L).NE.0.)  RS(L)=(RS(L)+ROOT)/2.
  650 CONTINUE
      GOTO 100
  600 WRITE(6,601) LIMA
  601 FORMAT(' D1.GT.0. IMPLIES TUMBLEHOME AT STATION ',I2)
      ADDMAS(LIMA)=0.
      GOTO 980
C     RS(L)  FOR A PARABOLIC SECTION OF PLATING
   18 DO 9 L=M1,N1
      E1=-TAN(Z(L))
      XX=(F1-B1+SQRT((B1-E1)**2-4.*A1*(C1-4.*R11)))/(2.*A1)
      ROOT=SQRT(XX**2+(E1*XX)**2)
      IF(RS(L).NE.0.)  RS(L)=(RS(L)+ROOT)/2.
      IF(RS(L).EQ.0.)  RS(L)=ROOT
    9 CONTINUE
  100 IF((K-11).GE.0.)  GOTO 110
      K=K+1
      GOTO 6
  110 RS(91)=4.*R11
      BEAMJ=RS(1)
C     DETERMINE AREA OF WET HULL*2
      SO=0.
      DANG2=.0043634
      RS(1)=RS(1)**2
      DO 104 J=1,90
      RS(J+1)=RS(J+1)**2
  104 SO=SO+DANG2*(RS(J)+RS(J+1))
      SO=SO*4.
C     NORMALIZE RS(L)
      DO 208 J=1,91
  208 RS(J)=RS(J)*PP/SO
```

```
      BE=SQRT(RS(1))
      SO=PP
  209 DO 210 J=1,8
      AJ=J
      E(J)=0.
      DO 220 I=1,91
      IF((J-1).NE.0) GOTO 211
      TE(I)=-ALOG(SQRT(RS(I)))*COS(2.*Z(I))
      GOTO 220
  211 TE(I)=((2.*AJ-1.)/(2.*AJ-2.))*COS(2.*AJ*Z(I))/RS(I)**(J-1)
  220 CONTINUE
      DO 212 L=1,90
  212 E(J)=E(J)+2.*(TE(L)+TE(L+1))*DANG
      DO 214 K=1,8
      AK=K
      TT=(2.*AJ-1.)*(2.*AK-1.)/(2.*(AJ+AK-1.))
      BI(J,K)=0.
      DO 213 I=1,90
  213 TA(I)=TT*COS(2.*(AJ-AK)*Z(I))/RS(I)**(J+K-1)
      TA(91)=TT*COS(PP*(AJ-AK))/RS(91)**(J+K-1)
            DO 214 L=1,90
      BI(J,K)=BI(J,K)+2.*(TA(L)+TA(L+1))*DANG
  214 CONTINUE
  210 CONTINUE
      DO 215 J=1,8
  215 BI(J,9)=E(J)
      DO 218 K=2,9
      DA=BI(K-1,K-1)
      IF(DA.NE.0.) GOTO 216
      K1=K-1
      GOTO 999
  216 DO 217 I=K,9
  217 BI(K-1,I)=BI(K-1,I)/DA
      DO 218 J=1,8
      IF((K-1).EQ.J) GOTO 218
      F=BI(J,K-1)
```

```
      DO 219 I=K,9
  219 BI(J,I)=BI(J,I)-F*BI(K-1,I)
  218 CONTINUE
      DO 228 I=1,8
  228 B(I)=BI(I,9)
      DO 247 I=1,91
      R(1)=SQRT(RS(I))
      CE(1)=-B(1)/R(1)
      D(1)=COS(Z(I))
      G(1)=SIN(Z(I))
      DO 245 J=2,8
      R(J)=R(J-1)*RS(1)
      CE(J)=-B(J)/R(J)
      AP=2*J-1
      D(J)=COS(AP*Z(I))
  245 G(J)=SIN(AP*Z(I))
      XI(I)=R(1)*D(1)
      ETA(I)=R(1)*G(1)
      DO 246 K=1,8
      XI(I)=XI(I)-CE(K)*D(K)
  246 ETA(I)=ETA(I)+CE(K)*G(K)
      RO(I)=SQRT(XI(I)**2+ETA(I)**2)
  247 Z(I)=ATAN2(ETA(I),XI(I))
      S1=SO
      DO 248 J=1,8
  248 S1=S1+B(J)*E(J)
      RM=0.
      W=.03661977
      DO 249 I=1,90
      DZ(I)=Z(I+1)-Z(I)
  249 RM=RM+DZ(I)*(RO(I)+RO(I+1))*.5
      RM=RM*W
      RM2=RM**2
  221 BETA=B(1)/RM2
      S1=S1/RM2
      DO 250 J=1,91
```

```
      DRO(J)=(RO(J)-RM)/RM
      DO 250 L=1,16
      P(L)=0.
      H=L*2
      T(J,L)=DRO(J)*COS(H*Z(J))
      IF(L.NE.1) GOTO 250
      TI(J)=T(J,1)*DRO(J)
  250 CONTINUE
      Q1=0.
      DO 251 J=1,16
      DO 251 L=1,90
      IF(J.EQ.1) Q1=Q1+DZ(L)*W*(TI(L)+TI(L+1))
  251 P(J)=P(J)+DZ(L)*W*(T(L,J)+T(L+1,J))
      SA=0.
      DO 254 K=2,16
      J1=K-1
      AL=2*K-1
      SA=SA+P(K)*P(J1)*AL
  254 CONTINUE
      CIJ=-(P(1)+SA-Q1)/2.
      S2=0.
      DO 260 J=1,16
      AJ=2*J-1
  260 S2=S2+AJ*P(J)**2
      S2=S1+S2*PP*RM2
      ROC22=S2/PP
C     DETERMINE CV
      CV=(2.*RM2*(ROC22-BETA-CIJ)-1.)/BE**2
C     DETERMINE ADDMAS FOR EACH SECTION
      ADDMAS(LIMA)=CV*BEAMJ**2*3.125*STASPA
  960 CONTINUE
      GOTO 965
  999 WRITE(6,998) K1
  998 FORMAT(1H1,'TERM NUMBER ',I11,' ON BI DIAGONAL IS ZERO.   THIS ABO
     1RTS THE ADMASS SUBROUTINE. ')
  965 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE HULL
      COMMON STA,STASPA,BEEM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SRJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
     1A(13,40),RAO(4,50),PERMOS(5,50)
      DIMENSION AREA(14),Y(14),SAREA(40)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
      WAREA=0.
C     INCREASE HULL THICKNESS BY 30%
      T1=T1*1.3
      T2=T2*1.3
      T3=T3*1.3
      DO 990 N=1,STA
C     MOMENT OF INERTIA SECTION
C     AREA OF PLATING
      AREA(1)=T3*SQRT(16.+DATA(2,N)**2)
      AREAS=AREA(1)
      DO 7 I=2,12
      IF(DATA(I+1,N).EQ.0.) GOTO8
      AREA(I)=T2*SQRT(16.+(DATA(I,N)-DATA(I+1,N))**2)
      AREAS=AREAS+AREA(I)
    7 CONTINUE
    8 I10=I-1
      I11=I
      I12=I+1
      TWL=I10*4.
      AREA(I11)=(DECKE(N)-TWL)*T2
      AREA(I12)=DATA(I11,N)*T1
      AREAS=AREAS+AREA(I11)+AREA(I12)
C     DETERMINE FIRST MOMENT OF THE PLATING
      AREA1=0.
      DO 9 J=1,I10
      Y(J)=2.+FLOAT((J-1)*4)
      AREA1=AREA1+AREA(J)*Y(J)
    9 CONTINUE
      Y(I11)=(DECKE(N)+TWL)/2.
```

```
      Y(I12)=DECKE(N)
      DO 11 J=I11,I12
      AREA1=AREA1+AREA(J)*Y(J)
   11 CONTINUE
C     DETERMINEHEIGHT OF THE NEUTRAL AXIS
      YNA=AREA1/AREAS
C     SUM TO GET PARTIAL I
      RMI(N)=0.
      DO 13 J=1,I12
      RMI(N)=RMI(N)+AREA(J)*(Y(J)-YNA)**2
   13 CONTINUE
C     ADD SEGMENT 2ND MOMENTS TO GET I(N)
      RMI(N)=RMI(N)/6.+(FLOAT((I11-2)*4**3)*T2+4.**3*T3+T2*(DECKE(N)-FLO
     1AT(I10*4))**3)/72.
C     J CALCULATION
C     DETERMINE AREAS BETWEEN WATERLINES
      AREA2=.333*(3.*DATA(2,N)-DATA(3,N))
      SARLA(N)=AREA2
      ARM2=(YNA-2.5)**2
C     DETERMINE 2ND MOMENTS ABOUT CENTROID AND NEUTRAL AXIS
      CENM=5.9*SQRT(DATA(2,N))
      CENNA=ARM2*AREA2
      SRJNA(N)=CENM+CENNA
      DO 15 J=2,I10
      IF(J.EQ.I10) AREA2=.333*(5.*DATA(J+1,N)+8.*DATA(J,N)-DATA(J-1,N))
      IF(J.NE.I10) AREA2=.333*(5.*DATA(J,N)+8.*DATA(J+1,N)-DATA(J+2,N))
      ARM2=(YNA-(4.*(DATA(J,N)+2.*DATA(J+1,N))/(3.*(DATA(J,N)+DATA(J+1,N
     1)))+FLOAT((J-1)*4)))**2
      B1=DATA(J+1,N)-DATA(J,N)
      B=DATA(J+1,N)
      CENM=1.778*(6.*B**2+6.*B*B1+B1**2)/(2.*B+B1)
      SRJNA(N)=SRJNA(N)+CENM+AREA2*ARM2
   15 SARLA(N)=SARLA(N)+AREA2
      H=DECKE(N)-FLOAT(I10*4)
      CENM=DATA(I11,N)*H**3/12.
      ARM2=((DECKE(N)+FLOAT(I10*4))/2.-YNA)**2
```

```
      AREA2=DATA(I11,N)*H
      SAREA(N)=(SAREA(N)+AREA2)*2.
      SRJNA(N)=(SRJNA(N)+CENM+AREA2*ARM2)*2.
C     DETERMINE TOTAL SECTION AREA
  989 WAREA=WAREA+SAREA(N)
C     DETERMINE KAG
  990 RKAG(N)=T2*DECKE(N)*1.807E08
C     DETERMINE AREA DENSITY
  991 DENSTY=DISPL*2240./(WAREA*32.2)
C     DETERMINE STATION MASS AND MASS MOMENT
      DO 17 N=1,STA
      SRJNA(N)=SRJNA(N)*DENSTY
      STAMAS(N)=SAREA(N)*DENSTY
   17 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NMODES
      COMMON STA,STASPA,BEAM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SKJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
     1A(13,40),PAO(4,50),PERMOS(5,50)
      DIMENSION OMEGA(3),ARLENE(3),VIRMAS(40),DIFF(40),AXX(40),BXX(40),C
     1XX(40),DXX(40),EXX(40),FXX(40),GXX(40),HXX(40),DIFF2(40)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
      LBP=FLOAT(STA-1)*STASPA
      BEAM=BEEM
      BD=BEAM/DEPTH
C     DETERMINE TAYLOR SHEAR CORRECTION TERM
   10 TAYLOR=3.5*DEPTH**2*(3.*BD**3+9.*BD**2+6.*BD+1.2)/((LBP**2*(3.*BD+
     11.)))
C     DETERMINE INITIAL OMEGA(1)
      OMEGA(1)=.2513E6*SQRT(RMI(STA/2)/(DISPL*LBP**3*(1.+BEAM/(2.*DRAFT)
     1)*(1.+TAYLOR)))-1.5
      OMEGA(2)=OMEGA(1)+1.
      OMEGA(3)=0.
      LOG=STA-1
C     DETERMINE VIRTUAL MASS REDUCTION FACTORS FOR EACH STATION
      DIFF(1)=ADDMAS(2)/2.
      DIFF(STA)=ADDMAS(STA-1)/2.
      ADMASM=ADDMAS(1)+ADDMAS(STA)
      DO 30 J=2,LOG
      ADMASM=ADMASM+ADDMAS(J)
      DIFF(J)=ABS((ADDMAS(J-1)-ADDMAS(J+1))/2.)
   30 CONTINUE
      DO 999 N=1,7
C     DETERMINE THE 3-D J FACTOR
   40 JFACT=1.02-3.*(1.2-1./FLOAT(N+1))*BEAM/LBP
      NODEF=FLOAT(N+1)*ADMASM/STA
      DIFFS=0.
      DO 45 J=1,STA
      DIFF2(J)=DIFF(J)+NODEF
      DIFFS=DIFF2(J)*ADDMAS(J)+DIFFS
```

```
   45 CONTINUE
      BBOY=(1.-JFACT)*ADMASM/DIFFS
      DO 50 J=1,STA
      VIRMAS(J)=STAMAS(J)+ADDMAS(J)*(1.-BBOY*DIFF2(J))
   50 CONTINUE
   60 DO 110 I=1,3
      IF(OMEGA(I).EQ.0.) GOTO 110
      OMEGA2=OMEGA(I)**2
C     PERFORM THE PROHL SEQUENCE
   70 AXX(1)=0.
      BXX(1)=0.
      CXX(1)=0.
      DXX(1)=-OMEGA2*SRJNA(1)
      EXX(1)=1.
      FXX(1)=LBP*.02
      GXX(1)=0.
      HXX(1)=1.
      DO 100 J=2,STA
      EI=2.160E9*(RMI(J)+RMI(J-1))
      KAG=(RKAG(J)+RKAG(J-1))/2.
      AXX(J)=AXX(J-1)+VIRMAS(J-1)*OMEGA2*EXX(J-1)
      BXX(J)=BXX(J-1)+VIRMAS(J-1)*OMEGA2*FXX(J-1)
      EXX(J)=EXX(J-1)+GXX(J-1)*STASPA-CXX(J-1)*STASPA**2/(2.*EI)+AXX(J)*
     1(STASPA**3/(6.*EI)-STASPA/KAG)
      FXX(J)=FXX(J-1)+HXX(J-1)*STASPA+DXX(J-1)*STASPA**2/(2.*EI)+BXX(J)*
     1(STASPA**3/(6.*EI)-STASPA/KAG)
      GXX(J)=GXX(J-1)+(AXX(J)*STASPA/2.-CXX(J-1))*STASPA/EI
      HXX(J)=HXX(J-1)+(BXX(J)*STASPA/2.+DXX(J-1))*STASPA/EI
      CXX(J)=CXX(J-1)-AXX(J)*STASPA+GXX(J)*OMEGA2*SRJNA(J)
      DXX(J)=DXX(J-1)+BXX(J)*STASPA-HXX(J)*OMEGA2*SRJNA(J)
  100 CONTINUE
      THETA=CXX(STA)/DXX(STA)
C     DETERMINE SHEAR EXCITATION AT THE STERN
      ARLNE(I)=AXX(STA)+BXX(STA)*THETA
  110 CONTINUE
  120 IF(OMEGA(1).EQ.0.) GOTO 160
```

```
C       IDENTIFY A ZERO IN THE SHEAR CURVE
  130 IF((ARLENE(1)*ARLENE(2)).GT.0.) GOTO 280
C       CHECK TO SEE IF OMEGA(2) IS A NATURAL FREQUENCY
      SPUD=ATAN2(ARLENE(1),ARLENE(2))
      SPUDS=ABS(SPUD)
      IF(ABS(SPUDS-1.570796).LE.0.0001) GOTO 250
C       DETERMINE INERMEDIATE OMEGA
  140 RATIO=ABS(ARLENE(1)/ARLENE(2))
      DELW1=(OMEGA(2)-OMEGA(1))*RATIO/(1.+RATIO)
      DELW2=OMEGA(2)-OMEGA(1)-DELW1
      OMEGA(3)=OMEGA(1)+DELW1
      OMEGA(1)=0.
      OMEGA(2)=0.
      GOTO 60
C       NARROW THE ZONE WIDTH
  160 IF(DELW1-DELW2) 162,162,164
  162 IF((ARLENE(1)*ARLENE(3)).LE.0.) DELDEL=ARLENE(3)*DELW1/(ARLENE(1)-
     1ARLENE(3))
      IF((ARLENE(1)*ARLENE(3)).GT.0.) DELDEL=ARLENE(3)*DELW1/ARLENE(1)
      GOTO 165
  164 IF((ARLENE(2)*ARLENE(3)).LE.0.) DELDEL=-ARLENE(3)*DELW2/(ARLENE(2)
     1-ARLENE(3))
      IF((ARLENE(2)*ARLENE(3)).GT.0.) DELDEL=-ARLENE(3)*DELW2/ARLENE(2)
  165 IF(DELDEL) 170,170,180
  170 OMEGA(1)=OMEGA(3)+2.*DELDEL
      OMEGA(2)=OMEGA(3)
      GOTO 190
  180 OMEGA(1)=OMEGA(3)
      OMEGA(2)=OMEGA(3)+2.*DELDEL
  190 OMEGA(3)=0.
C       CHECK FOR PROXIMITY TO NATURAL FREQUENCY
  200 IF(ABS(DELDEL)-.04) 270,270,60
  250 OMEGA(1)=OMEGA(2)
  270 FREQN(N)=(OMEGA(1)+OMEGA(2))/2.
      GOTO 500
  280 OMEGA(1)=OMEGA(2)
```

```
      OMEGA(2)=OMEGA(2)+1.
      GOTO 60
C     OBTAIN NATURAL MODE
  500 SUM=0.
      DO 510 J=1,STA
      MODE(N,J)=LXX(J)+FXX(J)*THETA
      SUM=SUM+MODE(N,J)**2*VIRMAS(J)
  510 CONTINUE
C     NORMALIZE MODE
      DO 520 J=1,STA
      MODL(N,J)=MODE(N,J)/SQRT(SUM)
  520 CONTINUE
C     ADVANCE SEARCH ZONE
      OMEGA(1)=OMEGA(2)+1.
      OMEGA(2)=OMEGA(2)+2.
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FORCE
      COMMON STA,STASPA,BEAM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SRJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
     1A(13,40),RAO(4,50),PERMOS(5,50)
      DIMENSION DMASS(40),BEAM(40),DELTA(40),G(2,40)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
C     DETERMINEDM'/DX
      DMASS(1)=ADDMAS(1)
      DO 20 J=2,STA
      DMASS(J)=ADDMAS(J)-ADDMAS(J-1)
      JUNE=J-1
      DMASS(JUNE)=(DMASS(JUNE)+DMASS(J))/2.
   20 CONTINUE
      DMASS(STA)=(DMASS(STA)-ADDMAS(STA))/2.
C     INITIALIZE PERMOS AND RAO
      DO 40 J=1,4
      DO 30 K=1,50
      PERMOS(J,K)=0.
      RAO(J,K)=0.
   30 CONTINUE
   40 CONTINUE
C     DETERMINE FREQUENCY LIMITS
      PERMOS(5,1)=SQRT(67.385/(FLOAT(STA-1)*STASPA))
      DOMEGA=(SQRT(67.385/STASPA)-PERMOS(5,1))/50.
      DO 42 J=2,50
      PERMOS(5,J)=PERMOS(5,J-1)+DOMEGA
   42 CONTINUE
C     LOOP FOR FORWARD SPEED ITERATION
      DO 910 L=1,4
      OMEGA=PERMOS(5,1)
C     SHIP SPEED IN FT/SEC
      VELOCE=FLOAT(L-1)*16.90
   44 M=1
   45 OMEGA2=OMEGA**2
      KAPPA=OMEGA2/32.174
```

```
C       DETERMINE WAVE HEIGHT CORRECTION TERM DELTA
        DO 60 J=1,STA
        SUM=0.
        LIMA=DATA(1,J)
        BEAM(J)=DATA(LIMA+1,J)
        DO 50 K=1,LIMA
        SUM=SUM+DATA(K+1,J)
   50 CONTINUE
        EXPON=KAPPA*DRAFT*SUM/(BEAM(J)*LIMA)
        IF(EXPON-50.) 55,55,58
   55 DELTA(J)=2.718**(-EXPON)
        GOTO 60
   58 DELTA(J)=0.
   60 CONTINUE
C       CUTOFF IF WAVE HEIGHTS ARE TOO SMALL
        SUM=0.
        DO 65 J=1,STA
        SUM=SUM+DELTA(J)
   65 CONTINUE
        IF(SUM-.01) 68,68,69
   68 M=50
        GOTO 870
   69 DO 70 J=1,STA
C       BOUYANCY, ADDED MASS, AND DAMPING WAVE FORCES
        G(1,J)=(124.74*BEAM(J)*STASPA-ADDMAS(J)*OMEGA2)*DELTA(J)
        G(2,J)=DELTA(J)*OMEGA*VELOCE*DMASS(J)
   70 CONTINUE
C       SUM MODAL EXCITATION FORCES
        DO 80 I=1,7
        SUM1=0.
        SUM2=0.
        SUM3=0.
        SUM4=0.
        DO 90 J=1,STA
        KX=KAPPA*STASPA*FLOAT(J-1)
        FORM1=MODE(I,J)*G(1,J)
        FORM2=MODE(I,J)*G(2,J)
```

```
      SUM1=SUM1+FORM1*COS(KX)
      SUM2=SUM2+FORM1*SIN(KX)
      SUM3=SUM3+FORM2*COS(KX)
      SUM4=SUM4+FORM2*SIN(KX)
   90 CONTINUE
C     MODAL DAMPING COEFFICIENT
      DELTAI=1.065E-02*SQRT(FREQN(I))
      DAMP1=2.*FREQN(1)*DELTAI/SQRT(39.478+DELTAI**2)
      PARTS1=SQRT(SUM1**2+SUM2**2)
      PARTS2=SQRT(SUM3**2+SUM4**2)
      IF(PARTS1+PARTS2-1.E-30) 80,80,100
  100 IF((ABS(SUM2)+ABS(SUM1)).LT.1.E-40) SUM2=1.E-40
      IF((ABS(SUM4)+ABS(SUM3)).LT.1.E-40) SUM4=1.E-40
  450 THETA1=ATAN2(SUM2,SUM1)
  470 THETA2=3.1416-ATAN2(SUM3,SUM4)
  500 RAO(L,M)=RAO(L,M)+SQRT((PARTS1*COS(THETA1)+PARTS2*COS(THETA2))**2+
     1(PARTS1*SIN(THETA1)+PARTS2*SIN(THETA2))**2)/((FREQN(I)**2-OMEGA2)*
     1*2+(DAMP1*OMEGA)**2)
   80 CONTINUE
      IF(L-1) 800,800,850
  800 OMEGAE=OMEGA
      GOTO 860
C     DETERMINE FREQUENCY OF ENCOUNTER
  850 OMEGAE=(-1.+SQRT(1.+.1243*VELOCE*OMEGA))/(6.22E-02*VELOCE)
  860 PERMOS(L,M)=8.305/OMEGAE**5*2.71828**(-.01555/OMEGAE**4)
  870 OMEGA=OMEGA+DOMEGA
  900 IF(M-50) 905,910,910
  905 M=M+1
      GOTO 45
  910 CONTINUE
      DO 940 L=1,4
      DO 950 J=1,50
      PERMOS(L,J)=SQRT(PERMOS(L,J))*RAO(L,J)*MODE(1,1)*12.
  950 CONTINUE
  940 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTPUT
      COMMON STA,STASPA,BEEM,DRAFT,DEPTH,T1,T2,T3,DISPL,RMI(40),RKAG(40)
     1,STAMAS(40),SRJNA(40),MODE(7,40),FREQN(7),ADDMAS(40),DECKE(40),DAT
     1A(13,40),RAO(4,50),PERMOS(5,50)
      INTEGER STA
      REAL MODE,KX,LBP,KAPPA,JFACT,KAG
      WRITE(6,770)
  770 FORMAT('1')
      WRITE(6,5)
    5 FORMAT(' STATION     MASS',9X,'J',10X,'I',9X,'KAG',6X,'ADDMAS'/)
      WRITE(6,10) (I,STAMAS(I),SRJNA(I),RMI(I),RKAG(I),ADDMAS(I),I=1,STA)
   10 FORMAT(3X,I2,4X,E10.3,E11.3,E11.3,E11.3,E11.3/)
      WRITE(6,770)
      DO 25 I=1,7
      WRITE(6,15) I,FREQN(I)
   15 FORMAT(1X,/' NATURAL FREQUENCY NUMBER ',I1,' IS ',F7.3,' RAD/SEC')
      WRITE(6,20) (MODE(I,J),J=1,STA)
   20 FORMAT(1X,5G12.4)
   25 CONTINUE
      WRITE(6,770)
      WRITE(6,30)
   30 FORMAT(22X,'V=0',21X,'V=10',20X,'V=20',20X,'V=30'/2X,'FREQUENCY
     1 RAO        RESPONSE       RAO        RESPONSE       RAO        RESPONS
     1E       RAO        RESPONSE'/)
      DO 40 M=1,50
      WRITE(6,35) PERMOS(5,M),(RAO(L,M),PERMOS(L,M),L=1,4)
   35 FORMAT(1X,9G12.4)
   40 CONTINUE
      RETURN
      END
```

```
20____52.63___80000._____100._____35._____55._____2.0_____1.75_____2.0
65._____60._____60._____55._____55._____55._____55._____55._____55._____
55._____55._____55._____55._____55._____55._____55._____55._____60._____

9.00 15.00 22.50 25.00 26.25 27.00 27.50 27.85 28.12 28.33 28.50 28.63 28.75
9.00 18.43 27.65 30.73 32.26 33.19 33.80 34.24 34.57 34.82 35.03 35.20 35.34
9.00 20.90 31.35 34.84 36.58 37.62 38.32 38.82 39.19 29.48 39.71 39.90 40.06
9.00 22.59 33.89 37.66 39.54 40.67 41.43 41.97 42.37 42.68 42.93 43.14 43.31
9.00 23.70 35.55 39.50 41.48 42.66 43.45 44.02 44.44 44.77 45.03 45.25 45.43
9.00 24.37 36.56 40.62 42.65 43.87 44.68 45.26 45.70 46.04 46.31 46.53 46.71
9.00 24.74 37.11 41.24 43.30 44.53 45.36 45.95 46.39 46.73 47.01 47.23 47.42
9.00 24.91 37.37 41.53 43.60 44.85 45.68 46.27 46.72 47.06 47.34 47.57 47.76
9.00 24.98 37.47 41.64 43.72 44.97 45.80 46.39 46.84 47.19 47.47 47.69 47.88
9.00 24.99 37.49 41.66 43.74 44.99 45.83 46.42 46.87 47.22 47.49 47.72 47.91
9.00 25.00 37.50 41.66 43.75 45.00 45.83 46.42 46.87 47.22 47.50 47.72 47.91
9.00 24.99 37.49 41.66 43.74 44.99 45.83 46.42 46.87 47.22 47.49 47.72 47.91
9.00 24.98 37.47 41.64 43.72 44.97 45.80 46.39 46.84 47.19 47.47 47.69 47.88
9.00 24.91 37.37 41.53 43.60 44.85 45.68 46.27 46.72 47.06 47.34 47.57 47.76
9.00 24.74 37.11 41.24 43.30 44.53 45.36 45.95 46.39 46.73 47.01 47.23 47.42
9.00 24.37 36.56 40.62 43.65 43.87 44.68 45.26 45.70 46.04 46.31 46.53 46.71
9.00 23.70 35.55 39.50 41.48 42.66 43.45 44.02 44.44 44.77 45.03 45.25 45.43
9.00 22.59 33.89 37.66 39.54 40.67 41.43 41.97 42.37 42.68 42.93 43.14 43.31
9.00 20.90 31.35 34.84 36.58 37.62 38.32 38.82 39.19 39.48 39.71 39.90 40.06
9.00 18.43 27.65 30.73 32.26 33.19 33.80 34.24 34.57 34.82 35.03 35.20 35.34
```

Sample Input

STATION	MASS	J	I	KAG	ADDMAS
1	0.215E 06	0.672E 08	0.272E 05	0.267E 11	0.150E 06
2	0.242E 06	0.642E 08	0.264E 05	0.247E 11	0.224E 06
3	0.274E 06	0.728E 08	0.293E 05	0.247E 11	0.286E 06
4	0.269E 06	0.596E 08	0.256E 05	0.226E 11	0.333E 06
5	0.282E 06	0.626E 08	0.267E 05	0.226E 11	0.366E 06
6	0.290E 06	0.643E 08	0.273E 05	0.226E 11	0.386E 06
7	0.294E 06	0.653E 08	0.277E 05	0.226E 11	0.397E 06
8	0.296E 06	0.658E 08	0.278E 05	0.226E 11	0.403E 06
9	0.297E 06	0.660E 08	0.279E 05	0.226E 11	0.405E 06
10	0.297E 06	0.660E 08	0.279E 05	0.226E 11	0.405E 06
11	0.297E 06	0.660E 08	0.279E 05	0.226E 11	0.405E 06
12	0.297E 06	0.660E 08	0.279E 05	0.226E 11	0.405E 06
13	0.297E 06	0.660E 08	0.279E 05	0.226E 11	0.405E 06
14	0.296E 06	0.658E 08	0.278E 05	0.226E 11	0.403E 06
15	0.294E 06	0.653E 08	0.277E 05	0.226E 11	0.397E 06
16	0.290E 06	0.643E 08	0.273E 05	0.226E 11	0.386E 06
17	0.282E 06	0.626E 08	0.267E 05	0.226E 11	0.366E 06
18	0.269E 06	0.596E 08	0.256E 05	0.226E 11	0.333E 06
19	0.248E 06	0.551E 08	0.239E 05	0.226E 11	0.286E 06
20	0.242E 06	0.642E 08	0.264E 05	0.247E 11	0.224E 06

Sample Output

```
NATURAL FREQUENCY NUMBER 1 IS   2.407 RAD/SEC
  0.5786E-03  0.4416E-03  0.3039E-03  0.1682E-03  0.3929E-04
 -0.7721E-04 -0.1761E-03 -0.2527E-03 -0.3030E-03 -0.3245E-03
 -0.3159E-03 -0.2778E-03 -0.2124E-03 -0.1230E-03 -0.1447E-04
  0.1081E-03  0.2392E-03  0.3738E-03  0.5080E-03  0.6394E-03

NATURAL FREQUENCY NUMBER 2 IS   6.040 RAD/SEC
  0.5409E-03  0.3244E-03  0.1062E-03 -0.9475E-04 -0.2534E-03
 -0.3465E-03 -0.3630E-03 -0.3040E-03 -0.1835E-03 -0.2633E-04
  0.1359E-03  0.2707E-03  0.3505E-03  0.3579E-03  0.2878E-03
  0.1486E-03 -0.4080E-04 -0.2555E-03 -0.4709E-03 -0.6699E-03

NATURAL FREQUENCY NUMBER 3 IS  10.611 RAD/SEC
  0.4966E-03  0.2150E-03 -0.6385E-04 -0.2806E-03 -0.3748E-03
 -0.3183E-03 -0.1375E-03  0.9863E-04  0.3022E-03  0.3981E-03
  0.3511E-03  0.1787E-03 -0.5520E-04 -0.2634E-03 -0.3667E-03
 -0.3225E-03 -0.1381E-03  0.1351E-03  0.4253E-03  0.6718E-03

NATURAL FREQUENCY NUMBER 4 IS  15.573 RAD/SEC
  0.4547E-03  0.1138E-03 -0.2053E-03 -0.3765E-03 -0.3153E-03
 -0.5812E-04  0.2399E-03  0.3993E-03  0.3232E-03  0.5536E-04
 -0.2454E-03 -0.4006E-03 -0.3191E-03 -0.5119E-04  0.2403E-03
  0.3773E-03  0.2712E-03 -0.3199E-04 -0.3896E-03 -0.6606E-03

NATURAL FREQUENCY NUMBER 5 IS  20.577 RAD/SEC
  0.4191E-03  0.2339E-04 -0.3101E-03 -0.3719E-03 -0.1116E-03
  0.2551E-03  0.4097E-03  0.2133E-03 -0.1662E-03 -0.3985E-03
 -0.2809E-03  0.8392E-04  0.3776E-03  0.3450E-03  0.1800E-04
 -0.3131E-03 -0.3558E-03 -0.6153E-04  0.3585E-03  0.6445E-03

NATURAL FREQUENCY NUMBER 6 IS  25.436 RAD/SEC
  0.3874E-03 -0.5765E-04 -0.3748E-03 -0.2773E-03  0.1468E-03
  0.4170E-03  0.2093E-03 -0.2404E-03 -0.4023E-03 -0.8450E-04
  0.3340E-03  0.3545E-03 -0.4845E-04 -0.3967E-03 -0.2796E-03
  0.1571E-03  0.3920E-03  0.1463E-03 -0.3332E-03 -0.6077E-03

NATURAL FREQUENCY NUMBER 7 IS  29.985 RAD/SEC
  0.3545E-03 -0.1260E-03 -0.3935E-03 -0.1221E-03  0.3489E-03
  0.3276E-03 -0.1733E-03 -0.4079E-03 -0.1972E-04  0.4000E-03
  0.2106E-03 -0.2989E-03 -0.3505E-03  0.1355E-03  0.4211E-03
  0.8066E-04 -0.3612E-03 -0.2413E-03  0.2888E-03  0.5959E-03
```

Sample Output

	V=0		V=10	
FREQUENCY	RAO	RESPONSE	RAO	RESPONSE
0.2596	4.285	0.4529	24.84	1.282
0.2770	5.968	0.7934	26.82	2.108
0.2945	8.018	1.218	28.30	2.960
0.3119	10.32	1.679	29.03	3.681
0.3293	12.77	2.130	28.79	4.142
0.3468	15.27	2.533	27.34	4.259
0.3642	17.72	2.861	24.49	3.991
0.3817	19.99	3.096	20.11	3.346
0.3991	21.95	3.227	14.32	2.388
0.4165	23.46	3.253	8.220	1.356
0.4340	24.40	3.176	8.562	1.383
0.4514	24.66	3.003	17.34	2.721
0.4689	24.17	2.748	28.26	4.288
0.4863	22.88	2.427	39.49	5.768
0.5037	20.81	2.059	49.95	7.001
0.5212	18.04	1.664	58.59	7.865
0.5386	14.70	1.265	64.37	8.266
0.5560	10.93	0.8828	66.38	8.145
0.5735	7.134	0.5359	63.90	7.488
0.5909	3.430	0.2410	56.58	6.331
0.6084	0.5410	0.3560E-01	44.57	4.763
0.6258	2.690	0.1659	28.67	2.925
0.6432	4.545	0.2631	10.81	1.054
0.6607	5.501	0.2993	11.73	1.093
0.6781	5.552	0.2841	29.24	2.605
0.6956	4.805	0.2316	43.73	3.728
0.7130	3.466	0.1575	52.54	4.288
0.7304	1.809	0.7762E-01	54.10	4.230
0.7479	0.1687	0.6840E-02	47.90	3.590
0.7653	1.280	0.4911E-01	34.66	2.492
0.7828	2.207	0.8016E-01	16.78	1.158
0.8002	2.545	0.3764E-01	8.525	0.5653
0.8176	2.307	0.7540E-01	25.43	1.621
0.8351	1.622	0.5036E-01	39.06	2.395
0.8525	0.7051	0.2082E-01	44.78	2.643
0.8699	0.2318	0.6514E-02	41.10	2.336
0.8874	0.8990	0.2406E-01	28.74	1.575
0.9048	1.215	0.3100E-01	11.57	0.6115
0.9223	1.142	0.2781E-01	13.62	0.6945
0.9397	0.7698	0.1790E-01	29.66	1.461
0.9571	0.2937	0.6528E-02	38.82	1.847
0.9746	0.3054	0.6492E-02	37.70	1.734
0.9920	0.5443	0.1107E-01	26.50	1.179
1.009	0.5498	0.1072E-01	10.05	0.4328
1.027	0.3514	0.6566E-02	15.68	0.6541
1.044	0.1576	0.2825E-02	30.52	1.234
1.062	0.2912	0.5009E-02	36.38	1.426
1.079	0.3596	0.5941E-02	30.60	1.163
1.097	0.2442	0.3877E-02	15.54	0.5731
1.114	0.5232E-01	0.7988E-03	10.28	0.3632

V=20		V=30	
RAO	RESPONSE	RAO	RESPONSE
52.31	1.039	79.86	0.5709
58.28	2.253	89.82	1.472
63.44	3.819	98.67	2.974
67.29	5.570	105.6	5.005
69.27	7.213	109.9	7.301
68.83	8.450	110.5	9.467
65.42	9.025	106.6	11.06
58.60	8.767	97.41	11.68
48.13	7.599	82.57	11.02
34.11	5.568	61.96	8.951
17.89	2.971	36.54	5.590
13.36	2.231	15.57	2.478
32.81	5.454	37.64	6.148
56.21	9.237	72.96	12.10
79.11	12.78	108.3	18.07
99.14	15.67	139.7	23.29
114.1	17.57	163.7	27.13
121.8	18.25	177.2	29.04
120.7	17.54	177.4	28.65
109.7	15.46	162.9	25.85
89.03	12.14	133.5	20.76
59.83	7.889	91.01	13.85
25.40	3.236	40.07	5.955
18.50	2.277	25.38	3.660
52.96	6.297	76.68	10.84
82.65	9.492	121.6	16.73
101.6	11.27	150.7	20.19
106.4	11.40	158.7	20.69
95.66	9.903	143.4	18.19
70.56	7.060	106.5	13.13
35.54	3.438	54.31	6.515
15.42	1.442	22.39	2.612
48.58	4.396	71.73	8.137
76.50	6.700	113.9	12.57
88.86	7.535	132.9	14.27
82.41	6.768	123.7	12.92
58.38	4.646	88.02	8.943
24.12	1.861	36.68	3.628
26.24	1.963	38.86	3.742
58.55	4.251	87.44	8.198
77.39	5.453	116.0	10.59
75.64	5.175	113.6	10.10
53.54	3.559	80.58	6.987
20.54	1.327	31.03	2.623
31.04	1.950	46.40	3.824
60.94	3.724	91.37	7.346
72.93	4.336	109.5	8.588
61.48	3.558	92.36	7.071
31.29	1.764	47.05	3.517
20.62	1.132	30.95	2.259

Sample Output

REFERENCES

1. Aertssen, G. and Ir.R.DeLembre, "Calculation and Measurement of the Vertical and Horizontal Vibration Frequencies of a Large Ore Carrier," Trans. NEC Institution of Engineers and Shipbuilders, Vol. 86, 1969-70, p.9-12.

2. Andersson, G. and K. Norrand, "A Method for the Calculation of Vertical Vibration with Several Nodes and Some Other Aspects of Ship Vbiration," Trans, RINA, 111, 1969, p. 367-383.

3. Bishop, R.E.D., "On the Strength of Large Ships in Heavy Seas," The South African Mechanical Engineer, Dec. 1971, p. 338-353.

4. Bishop, R.E.D. and W.G. Price, "A Note on Structural Damping of Ship Hulls," Journal of Sound and Vibration, 56(4), 1978, p. 495-499.

5. Bishop, R.E.D. and W.G. Price, The Dynamics of Marine Vehicle and Structures in Waves. London: Engineering Publications LTD, 1974.

6. Bishop, R.E.D. and W.G. Price, "On the Truncation of Spectra," International Shipbuilding Progress, Vol. 25, Jan, 1978, p. 3-6.

7. Bishop, R.E.D. and W.G. Price, "On the Relationship Between Dry Modes and Wet Modes in the Theory of Ship Response," Journal of Sound and Vibration, 45(2), 1976, p. 157-164.

8. Bishop, R.E.D. and W.G. Price, "Allowance for Shear Distortion and Rototary Inertia of Ship Hulls," Journal of Sound and Vbirations, 47(3), 1976, p. 303-311.

9. Bishop, R.E.D. and W.G. Price, "On Modal Analysis of Ship Strength," Proceedings of the Royal Society of London, A-341, 1974, p. 121-134.

10. Bishop, R.E.D. et al, "On the Structural Dynamics of Ship Hulls in Waves," Trans. RINA, 1973, p. 257-274.

11. Bishop, R.E.D. et al, "On the Dynamics of Slamming, "Trans. RINA, 1978, p. 259-269.

12. Bishop, R.E.D. et al, "On Wave Induced Stress in a Ship Executing Symmetric Motions," Philosophical Transactions of the Royal Society of London, A-275, 1973, p. 1-32.

13. Boyce, William E. and Richard C. DiPrima, Elementary Differential Equations. New York: John Wiley and Sons, 1969.

14. Comstock, John P. ed. Principles of Naval Architecture. New York: SNAME, 1967.

15. Cowper, G.R., "The Shear Coefficient in Timoshenko's Beam Theory," Journal of Applied Mechanics, 33, June 1966, p. 335-340.

16. Goodman, R.A., "Wave Excited Main Hull Vibration in Large Tankers and Bulk Carriers," Trans. RINA, 113, 1971, p. 167-184.

17. Jensen, J. Juncher and Niels FL. Madsen, "A Review of Ship Hull Vibration," Danish Center for Applied Mathematics and Mechanics.

18. Korvin-Kroukovsky, B.V. Theory of Seakeeping. New York: SNAME, 1961.

19. Kregszig, Erwin. Advanced Engineering Mathematics. New York: John Wiley and Sons, 1962.

20. Lamb, Sir Horace. Hydrodynamics. New York: Dover Publications, 6th ed., 1945.

21. Landweber, L. and M. Macagno, "Added Masses of Two Dimensional Forms by Conformed Mappings," Journal of Ship Research, 11(2), June 1967, p. 109-116.

22. Lewis, Frank M., "The Inertia of the Water Surrounding a Vibrating Ship," Trans. SNAME, 37, 1929, p. 1-20.

23. MacDuff, John N. and John R. Curreri, Vibration Control. New York: McGraw-Hill Book Company, Inc., 1958.

24. McGoldrick, R.T. and V.L. Russo, "Hull Vibration Investigation on SS Sopher Mariner," Trans. SNAME, 63, 1955.

25. Meirovitch, Leonard. Elements of Vibration Analysis. New York: McGraw-Hill Book Company, Inc., 1975.

26. Ochi, Michel K., "Wave Statistics for the Design of Ships and Ocean Structures," Trans. SNAME, 1979.

27. Robb, Andrew McCance. Theory of Naval Architecture. London: Charles Griffin and Company, Ltd., 1952.

28. Schade, Commodore H.A., "The Effective Breath Concept in Ship Structure Design," Trans. SNAME, 61, 1953, p. 410-430.

29. St.Denis, Manley and Willard J. Pierson, "On the Motions of Ships in Confused Seas," Trans. SNAME, 61, 1953, p. 280-357.

30. Stiansen, S.G. et al, "Dynamic Response of Large Great Lakes Bulk Carriers to Wave Excited Loads," Trans. SNAME, 85, 1978, p. 174-208.

31. Taylor, J. Lockwood, "Same Hydrodynamic Inertia Coefficients," Trans. INA, 1930.

32. Todd, F.H., Ship Hull Vibration. Belfast: University Press, 1961.

33. Towsin, R.L., "Virtual Mass Reduction Factors," Trans. RINA, 111, 1969, p. 385-397.

34. vanGunsteren, F.F., "Springing. Wave Induced Vibration," International Shipbuilding Progress, 17, 1970, p. 333-347.

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